NPS-57CO71111A

Naval Postgraduate School

THE APPLICATION OF HOLOGRAPHIC INTERFEROMETRY TO THE DETERMINATION OF THE FLOW FIELD AROUND A RIGHT CIRCULAR CONE AT ANGLE OF ATTACK

by

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December 1971

Approved for public release; distribution unlimited

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ABSTRACT

The successful application of holography to the study of three dimensional flow fields due to phase objects has been reported in the literature. The present report extends this technique to the study of density fields around opaque bodies as would normally be encountered in wind tunnel experiments. The density field around a 10 degree half-angle cone at zero and ten degree angle of attack has been investigated by means of the finite fringe holographic interferometry. The three dimensional density field obtained from the reduction of the interferograms was found to agree with that obtained from an analytical solution of the governing equations.

This report was supported by: Airtask No. A310310A/551A/1 R010-03-01

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INTRODUCTION

This is a report on holographic investigations directed towards the determination of complex three-dimensional density fields. The report consists of essentially Appendix I which includes the thesis of R. C. Jagota on the determination of density fields about a cone at angle of attack in a wind tunnel.

A paper based on this thesis, "Finite Fringe Holographic Interferometry Applied to a Right Circular Cone at Angle of Attack, by R. C Jagota and D. J. Collins, has been accepted for publication by the Journal of Applied Mechanics.

Further work on the determination of the density in corner flows was begun towards the end of the contract year. This work is directed at determining the flow in a wing fuselage junction at transonic speeds.

A preliminary program using a flat plate-fin in the Naval Postgraduate School wind tunnel has been started at Monterey. This preliminary program is designed to solve some of the holographic problems associated with the transonic test that are to be conducted at NSRDC. Further reports on this work will be forthcoming in December.

APPENDIX I

United States
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THESIS

THE APPLICATION OF HOLOGRAPHIC INTERFEROMETRY TO THE DETERMINATION OF THE FLOW FIELD AROUND A RIGHT CIRCULAR CONE AT ANGLE OF ATTACK

by

Ravi Chandar Jagota

December 1970

The Application of Holographic Interferometry to the Determination of the Flow Field Around a Right Circular Cone at Angle of Attack

by

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Submitted in partial fulfillment of the
requirements for the degree of

AERONAUTICAL ENGINEER

from the

NAVAL POSTGRADUATE SCHOOL
December 1970

ABSTRACT

The successful application of holography to the study of three dimensional flow fields due to phase objects has been reported in the literature. The present report extends this technique to the study of density fields around opaque bodies as would normally be encountered in wind tunnel experiments. The density field around a 10 degree half-angle cone at zero and ten degree angle of attack has been investigated in the Naval Postgraduate School supersonic wind tunnel. In these experiments, the finite fringe method for the production of interferograms has, for the first time, been applied to holographic interferometry. The density field obtained from the reduction of the interferograms was found to agree with that obtained from an analytical solution of the governing equations as reported by D. J. Jones in Reference 1. The computer program used for the reduction of the interferograms has been evaluated for the effect of the presence of the cone and the shock wave and has been found to yield stable and accurate results.

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I. INTRODUCTION

For over three decades, the Mach-Zehnder interferometer has been widely used and has come to be accepted as a standard tool for the measurement of the flow field around bodies. Until the advent of the laser, its use required extreme stability from vibrations and precision optical components. Even with a laser as a light source, the resulting interferograms are purely two-dimensional in nature in that they provide fringe information for only one direction of view through the wind tunnel. Furthermore, due to the limit imposed by existing camera shutter speeds, the interferogram obtained does not truly represent the instantaneous flow field around the body but rather is an average over the interval of shutter opening.

The advent of holography together with the development of the high power Q-switched laser has introduced a new dimension in the field of flow measurement. Apart from removing the need for precision optical quality components in the case of double exposed holograms, the use of holography has, for the first time, enabled a three-dimensional view of the flow field to be obtained from a single hologram. The high power of the laser and the Q-switch mechanism have together enabled film exposure times in the region of 20 nano-seconds to be attained thus, in effect, "freezing" the flow during the hologram production process.

Techniques for the application of holography to interferometry have been reported in Reference 1 (Heflinger et al, 1966) and in Reference 2 (Brooks et al, 1966). Holographic interferometry has been used to determine the three-dimensional asymmetric field produced by free-jet flow (Reference 3). In this report, a series of holograms placed in a 90 degree sector around the jet were obtained simultaneously from a single exposure. Interferograms were then obtained from the holograms at suitable angular orientations within a 90 degree field of view. This provided sufficient information to determine the entire flow field since the jet was planar symmetric. Reduction of the fringe data to density information was accomplished by expressing the density in a series of orthogonal polynomials and inverting the fringe-density integral equation by a computer program using a scheme reported in References 4 and 5.

In the present report, the applicability of holography interferometry has been extended to those three-dimensional density fields produced by opaque bodies as would be encountered in wind tunnel experiments. The experiments were conducted in the Naval Postgraduate School supersonic blowdown wind tunnel at a Mach number of 2.8 using a ten degree half-angle cone at zero and ten degrees angle of attack. In this case, the multiple hologram technique was not employed. Since the flow fields investigated were steady, the procedure adopted was to obtain holograms to cover various sectors of the flow field by rotating the cone within

the wind tunnel, this being a method that would be fully applicable to any conventional wind tunnel setup.

Whereas in an infinite fringe field, the fringe information is obtained in a discontinuous manner, that is, at the points of maxima and minima, the interferogram from a finite fringe field can be evaluated at any desired point quite readily. This is of particular significance in the case of density fields where the variations in density are small. In the present case, the total variation in fringe number was generally in the region of only one to two fringes. The finite fringe method was also selected since it is less sensitive to vibrations than is the infinite fringe field, a problem encountered in Reference 3. The finite fringe field was produced by a translation of the diffusing screen in the scene beam a distance of between .025" and .030" between the two exposures of the hologram. A vertical fringe field, obtained by horizontal translation of the diffuser, was chosen in spite of the greater effort required to reduce the interferograms at a particular section owing to the greater uniformity and clarity of the fringes thus obtained as compared to horizontal fringes. Fringe data obtained from the interferograms was reduced to density using the same computer program as used in Reference 3. The density field obtained experimentally was compared with the analytical solution reported in Reference 6, interpolating the results therein for the actual mach number of the experiments. Good correlation was achieved, thus demonstrating

the utility of holographic interferometry for normal wind tunnel experimentation.

II. EXPERIMENTAL APPARATUS

A. THE WIND TUNNEL

This investigation was conducted in the Naval Postgraduate School blow-down supersonic wind tunnel. The wind tunnel is a blowdown-to-atmosphere facility which has a test section 4 inches by 4 inches in cross-section and 6 inches in length. Interchangeable nozzles are available for producing nominal test section mach numbers of 4.0, 2.8, and 1.7. The nominal run time is 5 minutes at a mach number of 2.8 with a maximum stagnation pressure of about 105 pounds per square inch. For this series of experiments, the side walls of the wind tunnel were replaced by plexiglas walls two inches thick having a refractive index of 1.49 so that a complete view of the flow in the test section could be obtained.

B. THE EXPERIMENTAL SETUP FOR PRESSURE MEASUREMENT

Figure 1 is a schematic representation of the equipment arrangement used for pressure measurement. The static pressures on 16 out of a total of 30 orifices on the surface of the cone were measured by a Scanivalve (Model 48J4-1065 with a 0-10 psia. transducer) which was calibrated in inches of mercury vacuum. The measurements were recorded on a Honeywell 2106 twelve channel Viscicorder. The scanivalve was capable of scanning 15 ports per second. Photographs of the experimental setup are provided in Figures 2 and 3. The plexiglas side walls of the wind tunnel with the Scanivalve

mounted on top of the test section are shown in Figure 2. The power supply, control box and Viscicorder can be seen in Figure 3.

C. THE HOLOGRAPHIC ARRANGEMENT

A schematic of the holographic arrangement employed is shown in Figure 4 and photographs thereof are in Figures 5(a) and 5(b). The equipment was mounted on a table that rested on a portion of the floor that was vibrationally isolated from the rest of the building. The monochromatic light source used was a Korad K-1 pulsed ruby laser operating at a wavelength of 6943 Angstroms together with a Pockels cell Q-switching device. The resulting effective exposure time was about 20 nanoseconds and, with a cavity length of the laser of 37 cms., a coherence length of about 2 cms. was obtained. A side band geometry was employed to obtain the holograms of the flow field around the cone. A diffuse glass in the path of the scene beam was used to produce light field holograms, which because of the diffusion of the direction of the rays in the scene beam, produced an effective field of view in a hologram of approximately 16 degrees. A continuous wave helium-neon laser was used for alignment of the Q-switched laser and the optics. A Lauda constant temperature circulator Model N controlled by an electronic relay type R-10 was used in conjunction with a Culligan de-ionizer to maintain the laser head and output etalon at a constant temperature of 28.4 degrees Centigrade.

In Figure 4, details of the side band geometry employed can be seen. The reference beam was led on top of the wind tunnel by means of the two mirrors M and the size of the beam could be adjusted by means of lenses U. The positions of the diffuser plate and the two grids, one on each of the wind tunnel side walls, are also shown therein. In Figure 5(a), the Korad K-1 laser and the smaller alignment laser are shown together with the arrangement of the optical equipment. The diffuser plate mounted on the precision X-Y slide can also be seen. The holographic table and test section was completely enclosed in a wooden box to enable the experiments to be conducted in the daytime. This is shown in Figure 5(b). Also shown therein are the Korad K-1 laser power supply and shutter control assemblies, the Culligan de-ionizer and the Lauda constant temperature circulator.

In order to be able to view the cone at angle of attack always in a direction perpendicular to its axis when it was rotated in the wind tunnel, it was necessary to construct separate grids for each rotational angle at which the holograms were taken. The front grid used for the 82.5 degree rotational position of the cone is shown in position in Figure 6. The grid lines can be seen to be parallel and perpendicular to the cone axis for this particular angular position. The orientation of the grids was designed to be at the correct angle for the particular rotational position of the cone. Since, on rotation, the axis of the cone was not in a vertical plane, it was necessary to offset the two

grids with respect to each other in such a manner that on lining up the grids, the correct viewing position perpendicular to the cone axis was automatically obtained. The method of obtaining the orientations and the relative offset of the grids is shown in Figures 7(a) and 7(b). Since the cone spindle was bent at a distance of 0.7 inches from the base of the cone, the various rotational positions of the cone tip are indicated in the end view in Figure 7(a). The corresponding positions of the cone axis in the side and bottom views are also shown therein. The orientation of the grid lines for each rotational position of the cone as shown in Figure 7(b) were obtained from the positions of the cone axis in the side view for each particular case. In order to be normal to the cone axis, lines perpendicular to the orientation of the cone axis in the bottom view were transcribed, each of these representing a line of sight. Since the distance between the grids was 7 7/8 inches, the necessary amount by which the two grids were required to be offset for each rotational position of the cone tip were obtained as shown in Figure 7(b).

D. THE WIND TUNNEL MODEL

The cone model used is shown in Figure 8. The 10 degree semi-apex angle cone model was fabricated out of stainless steel and had pressure ports at three cross-sections, the angular position of the ports being so arranged as to lie on a ray from the apex of the cone. Since a fixed mount was

employed, provision was made to attach the cone to spindles bent to various angles to achieve the desired angle of attack. Stainless steel tubing of .036 inches internal diameter was used from the static pressure orifices on the surface of the cone up to the base of the model, the diameter of this being adequate to provide sufficient time response of the Scanivalve. Any fifteen out of the thirty ports could be connected at a time to the Scanivalve through the hollow cone spindle and mount, this restriction being imposed by the internal diameter of the spindle.

Rotation of the model about the horizontal axis of the spindle necessitated drilling 4 holes on each side of the wind tunnel side walls through which the model spindle could be unlocked from the mount and rotated. The plexiglas side walls of the test section and the holes therein can be seen in Figure 6. The collar on the model spindle containing radial holes (also visible in the figure) was used to rotate the cone by means of a spike arrangement inserted through one of the holes in the tunnel side walls. The angle of rotation was established by lining up graduations on the spindle sleeve with a graduation on the mount. The model and spindle used for the axisymmetric case can be seen in Figure 9. The pressure ports at the three stations and one of the pressure tubes from the cone passing through the hollow spindle are visible in the photograph. After the spindle was screwed into the cone, it was locked in position by means of the setscrew shown.

III. THEORETICAL CONSIDERATIONS

A. EVALUATION OF THE DENSITY FROM HOLOGRAPHIC INTERFEROGRAMS

1. The Basic Equation of Interferometry

The interferometry principle is based on the fact that two originally coherent rays of light will reinforce or annul each other at a screen on which both rays are projected depending on their relative phase there. The phase difference is directly a function of the difference in the lengths of the optical paths traversed by the two rays. If the optical path is changed by an amount $N\lambda$, where λ is the wavelength of the light and N is an integer, then the order of interference changes by an amount N, that is, a shift of N fringes occurs. The change in optical path can be related to the density through the index of refraction and the Gladstone-Dale constant. If the speed of light in a medium is represented by $\frac{c_o}{n}$ where c_o is the speed of light in vacuum and n is the index of refraction, the additional time required to traverse the test section due to a change in the index of refraction from n_1 to n_2 can be calculated to be equal to $\Delta t = \frac{L}{c_o}(n_2 - n_1)$ and thus the change in optical path obtained is $c_o \Delta t$, that is, $L(n_2-n_1)$. The fringe shift, g, which is equal to $\frac{\Delta L}{\lambda}$ is thus given by $L(n_2-n_1)$.

For a given substance and for a given wavelength of light, the index of refraction is a function of density. In the case of gases, since the speed of light varies only slightly

from that in vacuum, it can be represented very accurately by the first two terms of the Taylor series:

$$n = 1 + \beta \frac{\rho}{\rho_s} \tag{1}$$

where

ρ_s = reference density at 0°C, 760 mm. Hg.

β = dimensionless constant related to the Gladstone-Dale constant by:

$$K = \frac{\beta}{\rho_s}$$

The value of β for air at λ = 5893 Angstroms is 0.000292, the variation with wavelength being small. The fringe shift relation for a fixed difference in index of refraction between the two beams becomes:

$$J = \beta \frac{L}{\lambda} \left(\frac{\rho_2 - \rho_\infty}{\rho_s} \right) \tag{2}$$

If the density is variable in the test section, the net change in optical path is the integrated effect along the ray which gives the relationship:

$$J = \frac{\beta}{\lambda \rho_s} \int_0^L (\rho - \rho_\infty)\, ds = Q \int_0^L f(x, y, z_c)\, ds \tag{3}$$

where

$$f(x, y, z_c) = \frac{\rho(x, y, z_c)}{\rho_\infty} - 1 \tag{4}$$

$$Q = \frac{\beta \rho_\infty}{\lambda \rho_s} \tag{5}$$

ds = incremental distance along a ray

z_c = a plane of constant z.

Equation (3) is the basic equation which has to be inverted to obtain the unknown density from the known fringe shift values obtained from an interferogram.

2. The Integral Inversion

The integral inversion method used was first reported by C. D. Maldonado et al in 1965 (1965; Olsen, 1968) and subsequently used by R. D. Matulka (Reference 3) to calculate the density variation in an asymmetric free jet from the fringe numbers obtained from holographic interferograms. The procedure involves the representation of the function $f(x,y,z_c)$ by a complete set of orthogonal functions, with the unknown coefficients being evaluated by use of the orthogonality relationship between the set of functions. The set of functions employed have a region of orthogonality that covers the entire plane and have the property that they are "invariant in form" to a rotation of the co-ordinate system. The co-ordinate system used for the inversion is shown in Figure 10 where X and Y represent fixed laboratory co-ordinates and X' and Y' the co-ordinates in which the fringe number function is defined. These co-ordinates, therefore, rotate with respect to the fixed co-ordinates X and Y as the angle of view through the test section is varied.

In operator form, equation (3) can be represented as:

$$g(\xi, y', z_c) = T f(x, y, z_c) \qquad (6)$$

and the object of the procedure is to evaluate f, that is, to obtain:

$$f(x, y, z_c) = T^{-1} g(\xi, y', z_c) \tag{7}$$

where T^{-1} represents the inverse of T.

This inversion is achieved by utilizing a pair of orthogonal polynomials $U^{\pm m}_{m+2k}(\alpha x, \alpha y)$ and $H_{m+2k}(\alpha y')$ which are related by the transform relationship:

$$T\left\{U_{2k}(\alpha x, \alpha y)\, e^{-\alpha^2 x'^2}\right\} = \frac{e^{\mp i m \xi}}{[k!(m+k)!]^{1/2}} \frac{1}{2^{m+2k}} H_{m+2k}(\alpha y') \tag{8}$$

where:

$$H_{m+2k}(\alpha y') = e^{\alpha^2 y'^2}\left[d^{2k} d(\alpha x)^{2k}\right]\left[e^{-\alpha^2 y'^2} H_m(\alpha y')\right] \tag{9}$$

and H_m are the Hermite polynomials.

The function $f(x,y,z_c)$ is first expanded in a series of the polynomials U_{m+2k} as:

$$f(x, y, z_c) = \sum_{m=0}^{\infty} \sum_{k=0}^{\infty} \epsilon_m C^{\pm m}_{m+2k}(\alpha)\; U^{\pm m}_{m+2k}(\alpha x, \alpha y)\; e^{-(\alpha^2 x^2 + \alpha^2 y^2)} \tag{10}$$

where $\epsilon_m = \frac{1}{2}$ for m = 0 and 1 for m = 1,2,3,....., α is an arbitrary scale factor, and $C^{\pm m}_{m+2k}$ are the unknown expansion coefficients. In order to evaluate these coefficients, an expression for the function $g(\xi,y',z_c)$ is obtained in terms of the polynomials by transforming $f(x,y,z_c)$ in accordance with equation (6). Using the transform relation given by equation (8), the following expression is obtained:

$$g(\xi, y', z_c) = \sum_{m=0}^{\infty} \sum_{k=0}^{\infty} \epsilon_m \left[k!\,(m+k)!\, \alpha^{2(m+2k)} \right]^{-\frac{1}{2}} C_{m+2k}^{\pm m}(\alpha)\, e^{\pm i m \xi} H_{m+2k}(\alpha y') \, e^{-\alpha^2 y'^2} \qquad (11)$$

The expansion coefficients can then be evaluated by applying the following orthogonality relationship to Equation (11):

$$\int_{-\pi}^{\pi} e^{\pm i m \xi} e^{\mp i m \xi} d\xi \int_{-\infty}^{\infty} H_{m+2k}(\alpha y') \, H_{n+2\ell}(\alpha y') \, e^{-\alpha^2 y'^2} dy' =$$

$$\frac{2\pi}{\alpha}^{3/2} \left[(m+2k)!\,(n+2\ell)!\, 2^{m+2k} \alpha^{n+2\ell} \epsilon_{mn} \delta_{(m+2k)(n+2\ell)} \right] e^{\mp i m \xi} H_{\nu+2\beta}(\alpha y') \qquad (12)$$

that is, by taking the scalar product of equation (11) with $e^{\mp i m \xi} H_{\nu+2\beta}(\alpha y')$ and using equation (12), the coefficients of the series an equation (10) are obtained as:

$$C_{m+2k}^{\pm m}(\alpha) = \frac{\alpha}{2\pi}^{3/2} \left[\frac{\{k!\,(m+k)!\}^{1/2}}{(m+2k)!} \right] \int_{-\pi}^{\pi} g(y', \xi, z_c) \, H_{m+2k}(\alpha y') \, e^{\mp i m \xi} \, dy' d\xi \qquad (13)$$

Substitution of equation (13) into equation (10) results in the density variation being given by:

$$f(x, y, z_c) = \left(\frac{\alpha}{\pi}\right)^2 \sum_{m=0}^{\infty} \sum_{k=0}^{\infty} \epsilon_m \frac{\{k!\,(m+k)!\}^{1/2}}{(m+2k)!} e^{-(\alpha^2 x^2 + \alpha^2 y^2)} \cdot$$

$$Re\left[\int_{-\pi}^{\pi} \int_{-\infty}^{\infty} g(y', \xi, z_c) \, e^{-i m \xi} H_{m+2k}(\alpha y') \, dy' d\xi \right] U_{m+2k}^{m}(\alpha x, \alpha y) \qquad (14)$$

The functions $U_{m+2k}^{\pm m}$ employed are complex polynomials defined as:

$$U_{m+2k}^{\pm m}(\alpha x', \alpha y') = (-1)^k \left(\frac{\alpha}{\pi}^{1/2}\right) \left(\frac{k!}{(m+2k)!}\right)^{1/2} \left[\alpha^2 (x'^2 + y'^2)\right]^{m/2} \cdot$$

$$e^{\pm i m \varphi} L_k^m \left[\alpha^2 (x'^2 + y'^2)\right] \qquad (15)$$

where $\phi = \tan^{-1} \frac{y'}{x'}$ and L_k^m are the Associated Laguerre polynomials:

$$L_k^m(\alpha^2x^2+\alpha^2y^2) = \sum_{s=0}^{m} \frac{(m+k)!}{(k-s)!\,(m+s)!\,s!} \left[(-1)(\alpha^2x^2+\alpha^2y^2)\right]^s \qquad (16)$$

Substitution of equation (15) into equation (14) yields:

$$f(x,y,z_c) = \left(\frac{\alpha}{\pi}\right)^2 \sum_{m=0}^{\infty}\sum_{k=0}^{\infty} \frac{\epsilon_m (-1)^k k!}{(m+2k)!} (\alpha^2x^2+\alpha^2y^2)^{m/2} L_k^m(\alpha^2x^2+\alpha^2y^2) \cdot \left[B_{m+2k}^m(\alpha)\cos(m\phi) + D_{m+2k}^m(\alpha)\sin(m\phi)\right] e^{-(\alpha^2x^2+\alpha^2y^2)} \qquad (17)$$

where:

$$B_{m+2k}^m(\alpha) = \int_{-\pi}^{\pi}\int_{-\infty}^{\infty} g(y',\xi,z_c)\cos(m\xi) H_{m+2k}(\alpha y')\,dy'\,d\xi \qquad (18)$$

$$D_{m+2k}^m(\alpha) = \int_{-\pi}^{\pi}\int_{-\infty}^{\infty} g(y',\xi,z_c)\sin(m\xi) H_{m+2k}(\alpha y')\,dy'\,d\xi \qquad (19)$$

Equations (17), (18), and (19) are the basic equations used to obtain the density distribution of a completely asymmetric flow field.

3. The Numerical Procedure

The calculation of the unknown coefficients $B_{m+2k}^m(\alpha)$ and $D_{m+2k}^m(\alpha)$ in the series representation of $f(x,y,z_c)$ as given by equation (17) is not analytically possible in general since the function $g(y',\xi,z_c)$ representing the variation of fringe number is an experimentally measured quantity and thus not explicitly given as a function of the co-ordinates of a field point. If the analytical procedure is to be used to obtain the density from experimental observations of the fringe numbers, therefore, it is necessary to evaluate

the double integrals in equations (18) and (19) numerically. This is achieved by first noting that for every $g(y',\xi,z_c)$ encountered experimentally, there is a circular domain outside which the fringe distribution is zero so that it is possible to replace the limits of integration of $+\infty$ and $-\infty$ in equations (18) and (19) by finite values. The function $g(y',\xi,z_c)$ is then approximated by constant step values over elemental areas into which the domain is partitioned. This results in the B and D coefficients being represented by the double series:

$$B^{m}_{m+2k}(\alpha) = \sum_{i=0}^{I-1}\sum_{j=0}^{J-1} g(\xi_j+\Delta\xi_j, x_i+\Delta x_i)\int_{\xi_j}^{\xi_j+1}\cos(m\xi)\,d\xi\int_{x_i}^{x_i+1} H_{m+2k}(\alpha x)\,dx \qquad (20)$$

and

$$D^{m}_{m+2k}(\alpha) = \sum_{i=0}^{I-1}\sum_{j=0}^{J-1} g(\xi_j+\Delta\xi_j, x_i+\Delta x_i)\int_{\xi_j}^{\xi_j+1}\sin(m\xi)\,d\xi\int_{x_i}^{x_i+1} H_{m+2k}(\alpha x)\,dx \qquad (21)$$

Using the derivative formula for Hermite polynomials, the following equations result:

$$B^{m}_{m+2k}(\alpha) = \left[\frac{1}{2\alpha m}(m+2k+1)\right]\sum_{i=0}^{I-1}\sum_{j=0}^{J-1} g(\xi_j+\Delta\xi_j, x_i+\Delta x_i)$$

$$\cdot\left[\sin(m\xi_{j+1})-\sin(m\xi_j)\right]\left[H_{m+2k+1}(\alpha x_{i+1}) - H_{m+2k+1}(\alpha x_i)\right] \qquad (22)$$

$$D^{m}_{m+2k}(\alpha) = -\left[\frac{1}{2\alpha m}(m+2k+1)\right]\sum_{i=0}^{I-1}\sum_{j=0}^{J-1} g(\xi_j+\Delta\xi_j, x_i+\Delta x_i)$$

$$\cdot\left[\cos(m\xi_{j+1})-\cos(m\xi_j)\right]\left[H_{m+2k+1}(\alpha x_{i+1}) - H_{m+2k+1}(\alpha x_i)\right] \qquad (23)$$

Since it is not possible to sum over an infinite number of terms, equation (17) is replaced by the sum of a finite series:

$$f(x,y,z_0) = \left(\frac{\alpha'}{\pi}\right)^2 \sum_{k=0}^{K} \sum_{m=0}^{M} \epsilon_m (-i)^k \left[\frac{k!}{(m+2k)!}\right] (\alpha^2 x^2 + \alpha^2 y^2) \, L_k^m (\alpha^2 x^2 + \alpha^2 y^2)$$

$$\cdot \left[B_{m+2k}^{m}(\alpha) \cos(m\phi) + D_{m+2k}(\alpha) \sin(m\phi)\right] e^{-(\alpha^2 x^2 + \alpha^2 y^2)} \qquad (24)$$

The accuracy of the representation depends on the values of K, M, $\Delta\xi$ and Δx chosen and convergence of the series is influenced by the parameter α.

B. THE FLOW AROUND A RIGHT CIRCULAR CONE

1. Axisymmetric Case

The problem of the supersonic flow around a cone has been extensively studied by several investigators ever since it was first opened up by Busemann, Bourquard and Taylor and Maccoll. The basic approach to the problem is outlined below but a detailed description may be found in References 12, 13, 14 and 17.

Starting with the equations of motion of inviscid flow in three dimensions expressed in spherical co-ordinates:

$$\left.\begin{aligned}
&u\frac{\partial u}{\partial r} + \frac{v}{r}\frac{\partial u}{\partial \theta} + \frac{w}{r\sin\theta}\frac{\partial u}{\partial \phi} + \frac{1}{\rho}\frac{\partial p}{\partial r} - \frac{v^2+w^2}{r} = 0 \\
&u\frac{\partial v}{\partial r} + \frac{v}{r}\frac{\partial v}{\partial \theta} + \frac{w}{r\sin\theta}\frac{\partial v}{\partial \phi} + \frac{1}{\rho r}\frac{\partial p}{\partial \theta} + \frac{uv - w^2\cot\theta}{r} = 0 \\
&u\frac{\partial w}{\partial r} + \frac{v}{r}\frac{\partial w}{\partial \theta} + \frac{w}{r\sin\theta}\frac{\partial w}{\partial \phi} + \frac{1}{\rho r\sin\theta}\frac{\partial p}{\partial \phi} + \frac{uw + vw\cot\theta}{r} = 0
\end{aligned}\right\} \qquad (1)$$

and the equation of continuity:

$$\frac{\partial}{\partial r}(\rho r^2 u \sin\theta) + \frac{\partial}{\partial \theta}(\rho r v \sin\theta) + \frac{\partial}{\partial \phi}(\rho r w) = 0 \qquad (2)$$

the cylindrical symmetry of the cone allows one to eliminate the velocity component w and all terms dependant on ϕ. In addition, application of the condition that the flow be conical, that is, no variation of the flow parameters with r, reduces equations (1) to:

$$\left.\begin{aligned} \frac{du}{d\theta} - v &= 0 \\ \frac{dv}{d\theta} + u + \frac{1}{\rho v}\frac{dp}{d\theta} &= 0 \end{aligned}\right\} \qquad (3)$$

the first of these equations asserting that the flow must be irrotational. The equation of continuity similarly reduces to:

$$\frac{d}{d\theta}(\rho v \sin\theta) + 2\rho u \sin\theta = 0 \qquad (4)$$

Elimination of v between (3) and (4) yields:

$$\left\{\frac{d^2u}{d\theta^2} + u\right\}\frac{du}{d\theta} + \frac{1}{\rho}\frac{dp}{d\theta} = 0 \qquad (5)$$

Setting:

$$\frac{dp}{d\theta} = \frac{dp}{d\rho}\frac{d\rho}{d\theta} = a^2\frac{d\rho}{d\theta} \qquad (6)$$

the following ordinary differential equation is obtained:

$$\frac{d^2u}{d\theta^2} + u = \frac{a^2(u + v\cot\theta)}{(v^2 - a^2)} \qquad (7)$$

From the Bernoulli equation:

$$\int \frac{dp}{\rho} + \frac{1}{2}(u^2 + v^2) = \frac{1}{2} c^2 \tag{8}$$

and with assumption of a perfect gas satisfying the equation:

$$\frac{p}{\rho^\gamma} = k \tag{9}$$

the velocity of sound a can be expressed in terms of the velocity components as:

$$a^2 = \frac{1}{2}(\gamma - 1)(c^2 - u^2 - v^2) \tag{10}$$

The density, pressure and temperature are then found to vary as:

$$\begin{aligned} &(a)\ \frac{\rho}{\rho_c} = (c^2 - u^2 - v^2)^{\frac{\gamma}{\gamma-1}} \\ &(b)\ \frac{p}{p_c} = \left(\frac{\rho}{\rho_c}\right)^\gamma = (c^2 - u^2 - v^2)^{\frac{1}{\gamma-1}} \\ &(c)\ \frac{T}{T_c} = \left(\frac{a}{a_c}\right)^2 = (c^2 - u^2 - v^2) \end{aligned} \tag{11}$$

For the solution of the governing equation (7) to be physically meaningful, it is necessary to satisfy the boundary condition of no normal velocity at the cone surface and the jump conditions at the shock wave. The physical conditions on either side of a shock wave are given by the Rankine Hugoniot relations:

$$\left.\begin{aligned} \cos^2\alpha &= \frac{(\gamma-1) + (\gamma+1)\left(\frac{x}{y}\right)}{2\gamma\left(\frac{U}{a_1}\right)^2} \\ \tan\beta &= \frac{(\gamma-1) + (\gamma+1)\left(\frac{x}{y}\right)}{(\gamma-1)\left(\frac{x}{y}\right) + (\gamma+1)} \\ \left(\frac{z}{x}\right)^{\frac{\gamma-1}{\gamma}} &= 1 + \frac{\gamma-1}{4\gamma\cos^2\beta}\left\{(\gamma-1) + (\gamma+1)\left(\frac{y}{x}\right)\right\} \end{aligned}\right\} \tag{12}$$

where, referring to Figure 11,

α = Angle between the normal to the shock wave and the streamlines in front of it.

β = Angle between the normal to the shock wave and the streamlines immediately behind it.

ϕ = Angle between the streamline at any point and the radius vector from the vertex of the cone.

U = Freestream velocity.

a = Velocity of sound in the freestream.

$\frac{x}{y} = p_w/p_1$

$\frac{x}{z} = p_w/p_o$

p_1 = Pressure of the undisturbed gas in front of the shock wave.

p_w = Pressure immediately behind the shock wave.

p_o = Stagnation pressure behind the shock wave.

The boundary condition at the shock wave thus reduces to the requirement that the ratio (x/z) defined by the Rankine Hugoniot relations be equal to the pressure ratio (p/p_o) defined by equation (11a). This together with equations (12) defines the physical characteristics of the shock wave.

Many schemes have been employed in order to numerically integrate the governing differential equation and to satisfy the boundary conditions to yield the complete solution of the flow field. Results of some of these are tabulated in References 6 and 14. For the purposes of this report, the more recent results in Reference 6 have been used.

2. Flow Around Cones with Small Yaw

The governing differential equation for the flow around a yawed cone is developed on similar lines as that for axisymmetric flow with the difference that in this case, the velocity components, pressure and density are expressed as a series representation about the axial flow values. A detailed description of the procedure may be found in References 13, 15 and 17. An examination of the results reveals that although the flow is no longer irrotational, the flow field can still be regarded as conical. Furthermore, the shock wave in this case is still a cone of the same semi-apex angle as in the no-yaw case with the difference that the shock wave is itself yawed at an angle which is proportional to the angle of yaw of the cone with respect to the freestream.

In Reference 6, the method employed to obtain the flow properties is basically one of iteration. An estimate is made of the shape of the shock wave and this shape is improved in such a way that the normal velocity at the surface of the cone is made close to zero. The results are presented in the co-ordinate system shown in Figure 12. Values of u, v, w, $\frac{\rho}{\rho_\infty}$, and p at various azimuthal angles ϕ are given for various non-dimensional distances between the body and the chock wave. Such results are tabulated for cones of different half-cone angles, Mach number and angle of attack at which the flow can no longer be considered conical. For the case of a 10 degree half-angle cone,

this limiting angle of attack is 10 degrees at a Mach number of 3.0 and 11 degrees at a Mach number of 2.0.

IV. DESCRIPTION OF THE EXPERIMENT

A. NUMERICAL SIMULATION OF THE EXPERIMENT

The suitability of the computer program HOLOFER for handling cases with discontinuities in slope had been investigated by means of various test function inputs into the program in Reference 3. The accuracy of the inversion in these cases was reported to be within 3.8 percent. In order to verify the applicability of the program to the present investigation, known values of the density along one line of sight, obtained from Reference 6 at a particular cross-section in the flow, were fed in as input and Mode 1 operation of the computer program utilized. In this mode, the program calculates the fringe number array that would result from the specified density distribution and then uses this data as the input for inversion to obtain the density distribution. One of the primary objectives of this investigation was to determine the effect of opaque objects introduced into the flow field and the presence of the shock wave on the resulting inversion. It was found that the effect of the presence of the cone could be reduced by the introduction of a fictitious density distribution in the area occupied by it of a constant value equal to the density at the cone surface.

B. EXPERIMENTAL PROCEDURE

1. Laboratory Techniques

It has been shown in Reference 10 that the transmitted intensity from a diffuse glass falls off below 30 percent of the incident intensity beyond viewing angles of ±8 degrees. Thus this represents a limit on the field of view obtainable from a single hologram. For the flow around a right circular cone at incidence, there is planar symmetry and hence a 90 degree field of view is required for complete coverage of the total flow field. This coverage can be obtained by the simultaneous production of a number of holograms in various azimuthal positions as employed in Reference 3 or by a rotation of the viewing angle between each production of a single hologram. Taking advantage of the steadiness of the flow fields investigated, holograms covering various sectors of the flow field were obtained by rotating the cone to six positions in a 90 degree sector, one hologram being obtained in each case.

Complete angular coverage of the flow field was obtained by a rotation of the cone and spindle arrangement in the mount by an angle of 15 degrees between each hologram. Since it was desired to obtain the flow field at the same cross-section for the various angles of rotation, it was necessary to obtain the holographic interferograms in such a way that the line of sight was always perpendicular to the axis of the cone. For this purpose, a pair of grids were used, one on each wall of the test section, in which

the lines were designed to be parallel and perpendicular to the cone axis for a particular rotational position of the cone. The two grids were offset laterally with respect to each other in order to account for the rotation of the cone axis away from the central plane when the cone was rotated. On production of the holograms, the variable viewing property of light field holograms was utilized to line up the two grids thereby ensuring that the line of sight was normal to the axis of the cone. This would have involved a considerable amount of complexity and effort if a conventional Mach-Zehnder interferometer had been employed for the same purpose.

Since the coherence length of the pulsed laser was only about 2 cms. at the output used, it was necessary to maintain the optical path lengths in the reference and scene beams nearly equal. Since the scene beam path traversed 4 inches of plexiglas while the reference beam passed through two thick plano-convex lenses, it was necessary to compensate for this by making the reference beam path 3.5 inches longer than that of the scene beam. A reference to scene beam ratio of 4:1 was found to yield good holograms.

The holograms were obtained on Agfa-Gevaert 8E-75 holographic plates, 4" x 4" in size. Development was for 5 minutes in Kodak D-19 developer, thirty seconds in an acetic acid stop bath of standard dilution, five minutes in standard fixer, one minute water wash followed by immersion in a Kodak PhotoFlo wetting agent prior to drying. Reconstruction

of the scene was achieved by illuminating the hologram with the light from a helium-neon laser operating at a wavelength of 6328 Angstroms.

2. Photographic Techniques

To obtain reconstruction of the image from the hologram, various methods can be employed. In the usual method where the hologram is re-illuminated by a beam similar to the original reference beam as shown in Figure 13, each point on the photograph is produced by a series of non-parallel rays that appear to originate from various sources on the diffuse glass that was used to construct the hologram. An almost parallel set of rays can be selected by the positioning of a small aperture at the focal plane of the imaging lens as shown in Figure 14. A similar effect is achieved by illuminating the hologram by a conjugate beam of small diameter as depicted in Figure 15. Because of the small diameter of the reconstruction beam, the illuminated portion of the hologram represents a small aperture and thus the image produced has a large depth of field. This was of considerable advantage in the production of the interferograms from the holograms since it enabled the front and rear grids, the cone and the fringes to be simultaneously projected on a screen and thus made the task of obtaining the correct viewing angle and the interferogram quite easy to achieve. A photograph of the scene was obtained by focusing the camera at the desired plane in the test section, usually in such a manner so as to be in the plane of the fringes.

The lines of sight recorded on the photograph in this case represent the almost parallel pencil of rays from the diffuser that pass through the effective aperture in the hologram.

3. Data Reduction

To obtain photographic interferograms, the hologram was illuminated by the conjugate reference beam using a continuous wave laser operating at a wavelength of 6328 Angstroms. A camera back with viewing screen was placed in the position of the real image as shown in Figure 13. By lining up the appropriate horizontal and vertical lines of the front and rear grids, the viewing angle corresponding to the particular line of sight desired was achieved. As the spacing of the lines on each grid was ½ inch and the two grids were physically 7 7/8 inches apart, a viewing angle of 3°38' was obtained by lining up a line on one of the grids with the adjacent one on the other. A f7.7 lens with full aperture and an exposure time of 4 seconds was used to obtain workable holograms on Poloroid Type 55 P/N film. The reduction of fringe shift was obtained by projecting the negative in a photoenlarger and by tracing the fringes, the cone surface, the shock wave and the grid line at the desired station at which the reduction was to be performed, on a sheet of paper. Some judgement was usually necessary in locating the centre of the fringes but in general the light fringes were found to be narrower than the dark fringes and a location of the fringes to within ±0.1 of the fringe spacing in the freestream could be obtained. The

fringes in the region outside the shock were extended towards the cone surface and the fringe shift read at the points of intersection of the displaced fringes with a number of rays from the cone vertex. The assumption of locally conical flow in the region within two to three fringes on either side of the section was made in order to obtain the fringe displacements at the section from those at the points of intersection of the rays from the cone vertex with the fringes on either side of the section. The distance between the cone tip and the section at which reduction was performed was measured from the projected image and, since the actual distance between these points was known, it was possible to calculate the factor by which the projected image was magnified as compared with its true size. This magnification factor was then used to determine the actual radial distances from the cone axis at which the fringe shifts were obtained. The radius of the inversion circle was selected such that the maximum distance of the shock from the cone axis was at 95 percent of the radius of the inversion circle. From the data so obtained, the radial variation of fringe number was plotted and a smooth curve drawn through the points. The fringe number at 201 equidistant points was then read off from the curve and utilized as the input into the computer program HOLOFER in Mode 3. This mode utilizes raw data taken at the proper interval and directly read into the G array in the program by Subroutine READ. Further details about the use of this computer

program are outlined in Appendix C. For the axisymmetric case, it was necessary to feed in the fringe data along one line of sight only and to obtain the density variation along one radial line. For the asymmetric case, fringe data was fed in along six lines of sight in a 90 degree field of view and the inverted density field obtained along 9 radial lines spanning a 180 degree field of view on one side of the axis of planar symmetry of the flow. A typical reduction of an interferogram to obtain the radial variation of fringe number at a section is shown in Appendix A.

V. EXPERIMENTAL RESULTS AND DISCUSSION

A. DENSITY FIELD OBTAINED EXPERIMENTALLY

1. Computer Simulation Results

The density distribution obtained from the AGARD 167 tables for the axisymmetric case was used as a test function in Computer program HOLOFER. The fringe distribution calculated from this density distribution was inverted to obtain the original density distribution. A plot of the original density distribution and that inverted by the program is shown in Figure 16. Since the Mach number for this inversion was 2.84, it was necessary to first interpolate the 13 values obtained from Reference 6 for a Mach number of 2.84, and then to obtain from these, the values at 201 equidistant points along a line of sight within the region of the inversion circle. This was done using Computer Program 1. As a side benefit obtained from a use of this program, the discontinuities in slope in the density distribution at the cone surface that would otherwise have resulted by assuming a constant value in the region occupied by the cone, were eliminated. Various values of K, M and interval size were experimented with to obtain the minimum deviation of the inverted density distribution from the input value in the region close to the shock wave. Increasing the value of K above 500 was found to have little effect on the accuracy, the maximum variation of the inverted density being 1.5

percent of the input density value at the point of inversion. The value of α was found to affect the total number of terms in the series representation of the B and D coefficients required for convergence within the value of ε specified. A value of K = 2000, M = 5, α = 2.5 and an interval size of 0.01 was found to yield the minimum effect of the shock discontinuity.

2. Experimental Results

a. Axisymmetric Case

A photograph of the holographic interferogram obtained is shown in Figure 17(a). The fringe data obtained from a reduction of the two sides of the cone is shown in Figure 17(b). Since the flow was axisymmetric, an average of the two readings was taken and a curve plotted. The values at 201 equidistant points was read off, this being input into Subroutine READ of computer program HOLOFER as the G array. The density distribution obtained from the output of the inversion is shown in Figure 18. Also shown plotted is the density field obtained from the AGARD 167 tables interpolated for a Mach number of 2.84 by computer program 2, this being the Mach number of the tunnel obtained from pressure measurements. The pressure data obtained from the viscicorder is shown in Figure 19. The value of Mach number calculated from these pressure measurements compares favorably with the value obtained in earlier experiments from shadowgraphs of the flow in the tunnel. A comparison of the density distributions from the AGARD tables

and those obtained from the experiment show good agreement, the maximum variation in overall density being 8 percent of the theoretical density value.

b. Asymmetric Case

The views along which holograms were taken for the cone at 10 degrees angle of attack are shown in Figure 20 and photographs of the interferograms obtained are shown in Figures 21 to 26. Except for the 67.5 degree and 82.5 degree angles of view, the perturbation to the flow on the leeward side of the cone was insufficient to enable either the shock position or the fringe displacements on this portion of the flow field to be obtained. The fringe shifts in this region were, therefore, taken as being zero. A comparison of the shock wave position obtained experimentally from the interferograms with that from the AGARD 167 tables interpolated for a Mach number of 2.84 is shown in Figure 27. The fringe data were input into computer program HOLOFER starting with the 82.5 degree fringe distribution in order to be consistent with the plane of symmetry for which that program was written. The resulting density distribution output from the program was along the lines of sight indicated in Figure 28. A comparison of the experimentally obtained radial distributions with those from the AGARD 167 tables interpolated for a Mach number of 2.84 are shown in Figures 29 to 35.

B. ERROR ANALYSIS

The errors in the final solution are mainly due to errors in the fringe data input into the computer program. One of the major sources of error was due to the difficulty in obtaining the slope of the fringe lines in the freestream to a very high degree of accuracy. From the results of the experiments conducted on a 10 degree half angle cone in Reference 18, it was apparent that the flow in regions close to the tip of the cone would be affected by the boundary layer on the cone whereas sections near the base of the cone would be affected by the wake and expansion of the flow. In order to obtain a flow which was locally nearly conical, it was thus necessary to reduce the interferograms at a section approximately 60 to 70 percent of the cone length downstream of the tip. At this section, however, the shock wave was about 1 inch from the axis of the cone and considering that the fringes in a region about ¼ inch adjacent to the tunnel walls were affected by the boundary layer, this left a region of about 3/4 inch in which the fringes of the flow in the freestream were present. It was thus difficult to obtain the slope of the fringes very accurately and it is estimated that errors of approximately ±1 mm. in the fringe data along any fringe could have resulted. However, since the fringe data along the section was obtained by interpolation between different fringes at varying radial distances from the cone axis, this would have resulted in a smoothing of the errors to about ±0.5 mm. Since the maximum fringe shifts obtained

were in the region of 8 mm., an error of ±0.5 mm. corresponds to an error of approximately ±8 percent in the magnitudes of the fringe shifts obtained.

The errors between the experimental and theoretical density curves in Figure 18 reflect the above estimates of error quite well. The experimentally obtained density distribution is generally about 5% higher than the theoretical curve and this rises to a maximum of 8% close to the cone surface. Similar errors are apparent in general in Figures 29 to 35. However, in these curves the maximum error is somewhat higher probably due to the fact that reduction was performed at a section closer to the base of the cone than for the axisymmetric case. In so far as the general magnitude of the density is concerned, the experimentally obtained distribution follows the theoretical one fairly well. Except for the density distributions for $\phi = 0°$ and $\phi = 22.5°$, the density starts to rise above the freestream value at a normalized radial distance from the cone axis of about 0.7, then assumes a nearly constant value before rising steeply again at a radial distance closer to the cone axis. This appears to be inconsistent with the data input into the program in which the radial distance of the shock from the cone axis is seen to decrease with increasing values of ϕ.

V. SUMMARY

A. CONCLUSIONS

The finite fringe method for the production of holographic interferograms has been applied successfully to the determination of the three-dimensional density distribution of the flow around a 10 degree half angle cone. The method has been demonstrated to yield results within an accuracy of 8 percent for the axisymmetric case and within 13 percent for the asymmetric case. The magnitude of the errors are mainly a result of the constraints imposed by the existing wind tunnel size and the fact that the cone model used produced perturbations in the flow field of a fairly small magnitude so that small errors in measurement were reflected as relatively large percentage errors. The computer program HOLOFER has been evaluated for the effect of the shock and the cone and found to yield good results.

B. ACKNOWLEDGEMENTS

The writer wishes to gratefully acknowledge Dr. D. J. Collins for his most valuable guidance and assistance throughout the course of this investigation; the technical staff of the Department of Aeronautics under R. Besel and T. Dunton, particularly N. Leckenby, G. Gulbranson, G. Middleton, G. Teaby for the fabrication and assembly of the models and equipment used in these experiments and

D. Lawrence for the preparation of the necessary drawings; and Mrs. C. Roper for her considerable effort in the final preparation and typing of this report.

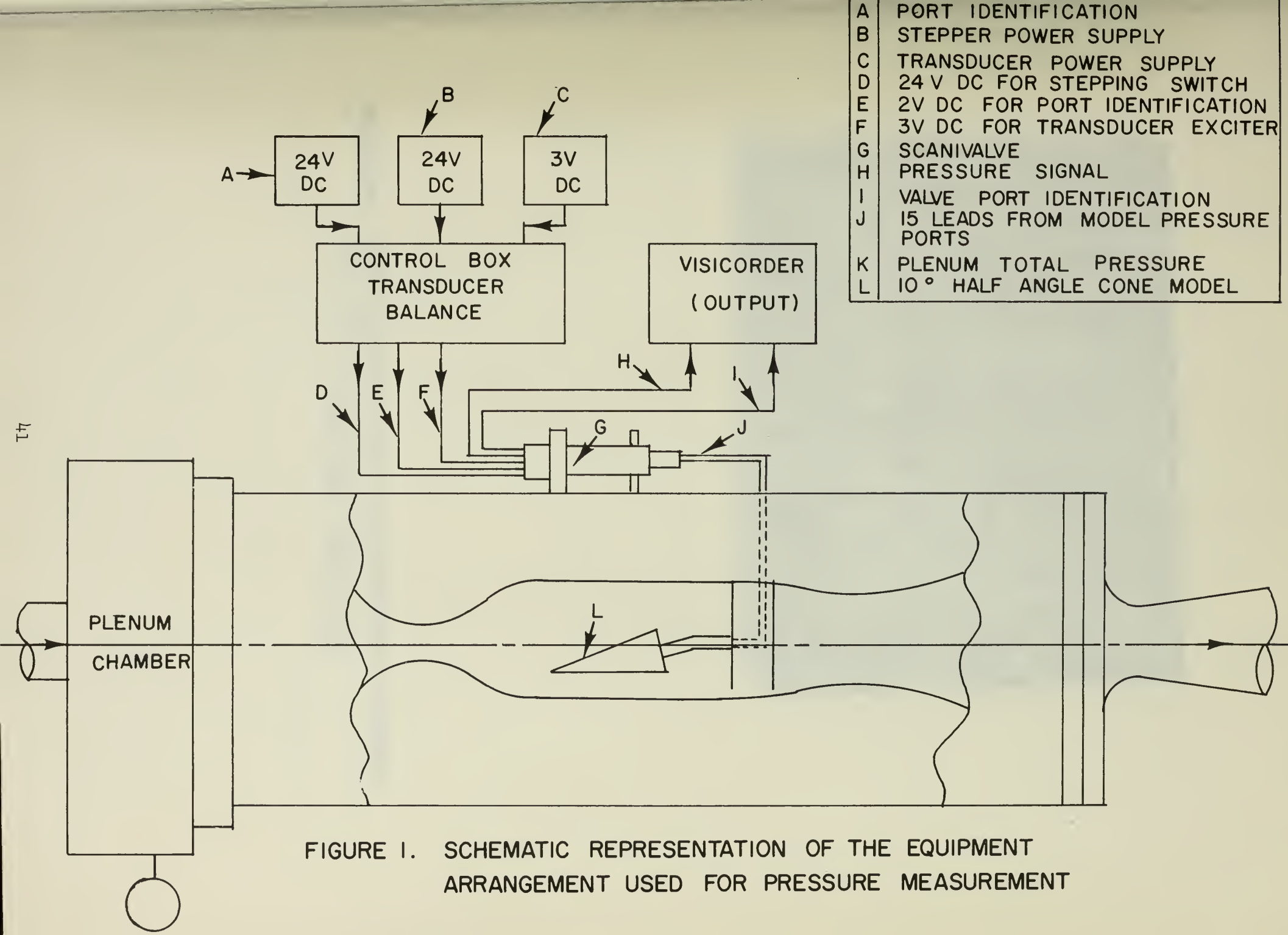


FIGURE 1. SCHEMATIC REPRESENTATION OF THE EQUIPMENT ARRANGEMENT USED FOR PRESSURE MEASUREMENT

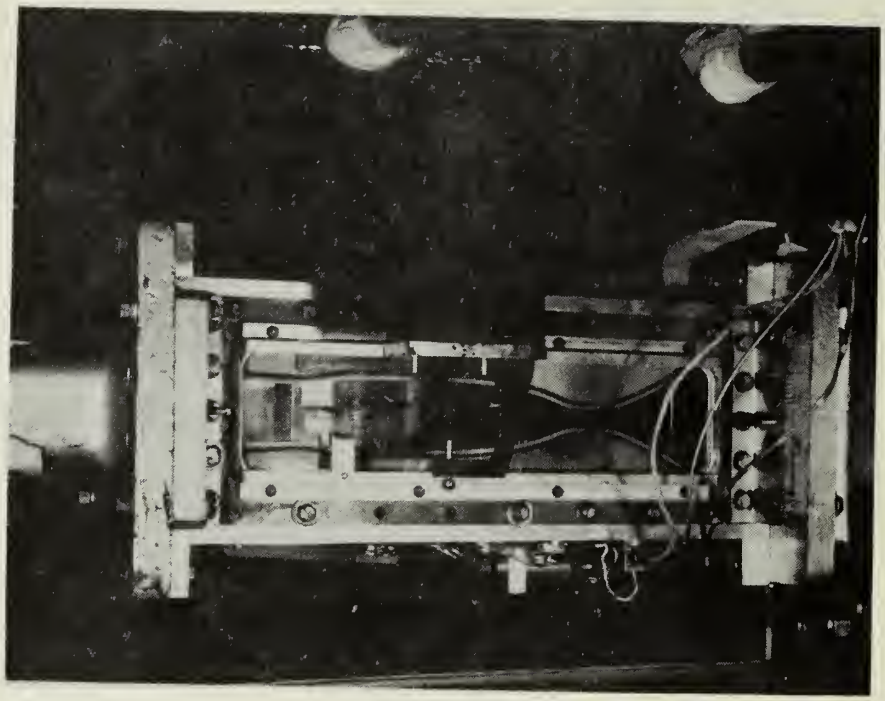


FIGURE 2. WIND TUNNEL TEST SECTION AND PRESSURE MEASURING EQUIPMENT

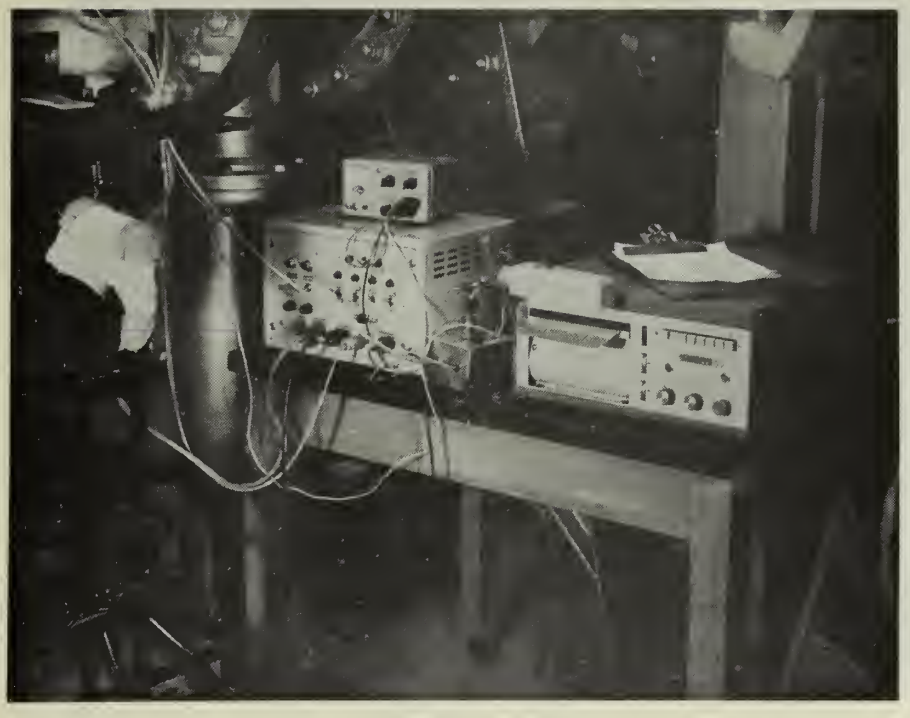
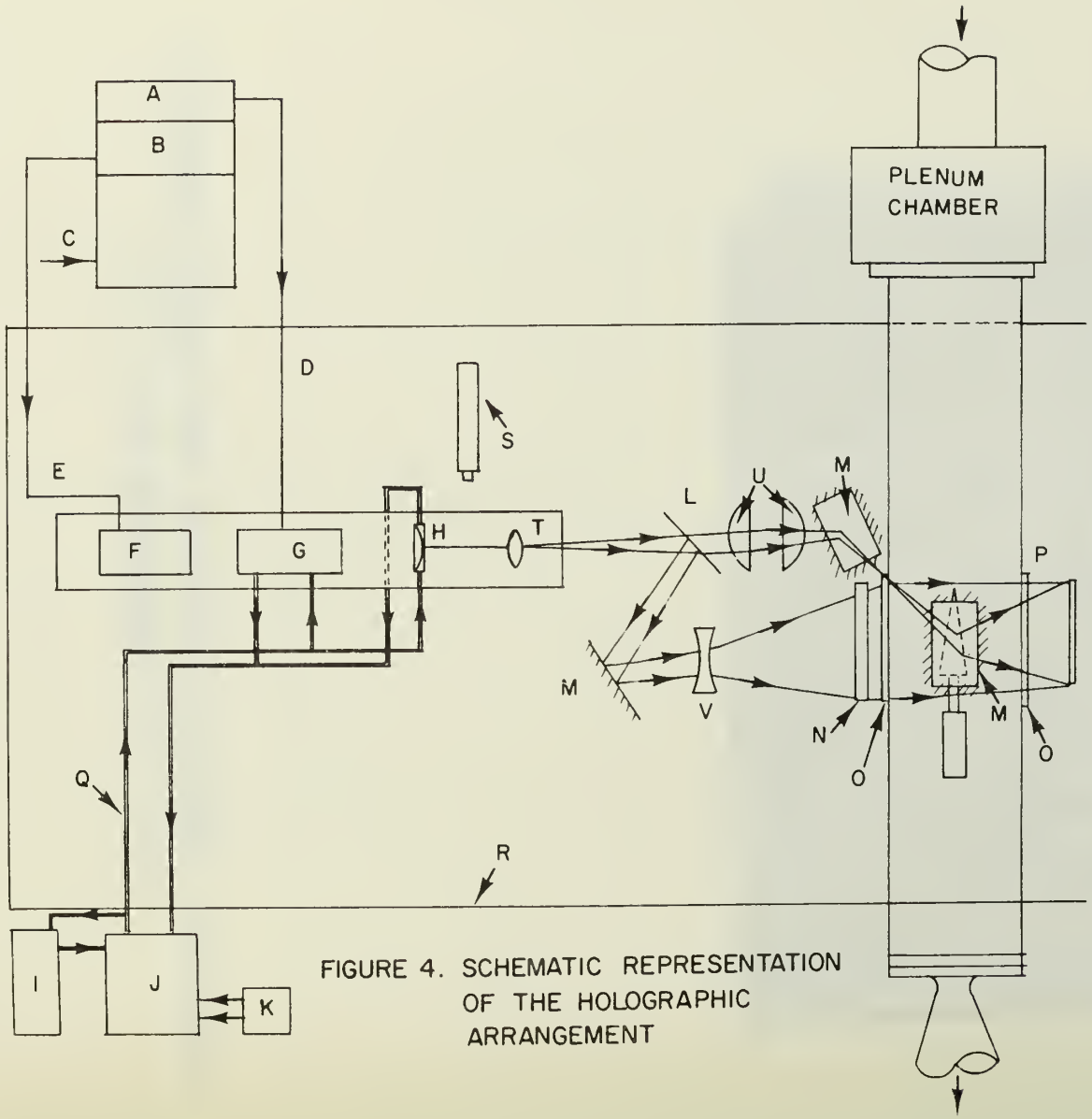


FIGURE 3. PRESSURE RECORDING EQUIPMENT, CONTROL BOX, AND POWER SUPPLIES



A	LASER HEAD POWER SUPPLY
B	SHUTTER ELECTRONICS
C	110V AC POWER SUPPLY
D	POWER SUPPLY TO FLASH LAMP
E	40 KV PULSE TO POCKELS CELL
F	POCKEL'S CELL
G	LASER HEAD
H	ETALON
I	DE-IONISER
J	CONSTANT TEMP WATER CIRCULATOR
K	ELECTRONIC RELAY CONTROL FOR CIRCULATOR
L	BEAM SPLITTER
M	FRONT SURFACE MIRROR
N	DIFFUSER PLATE
O	ALIGNMENT GRID
P	HOLOGRAM PLATE
Q	WATER SUPPLY TO LASER LEAD AND ETALON
R	HOLOGRAPHIC TABLE
S	HE-NE ALIGNMENT LASER
T	DOUBLE CONCAVE LENS F = 4.1 CM
U	PLANO-CONVEX LENS
V	DOUBLE CONCAVE LENS

FIGURE 4. SCHEMATIC REPRESENTATION OF THE HOLOGRAPHIC ARRANGEMENT

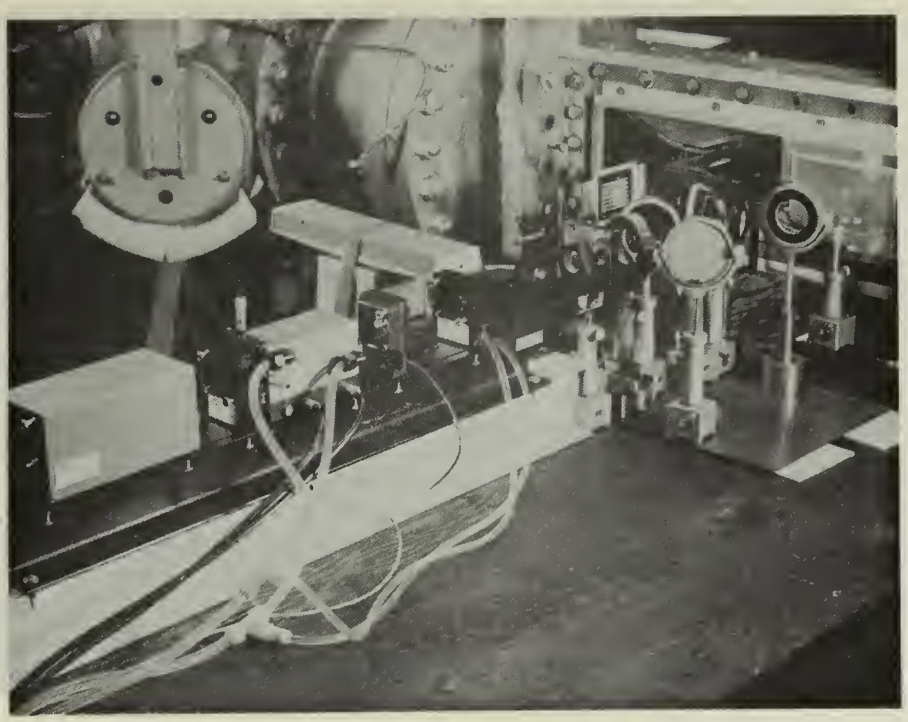


FIGURE 5a. HOLOGRAPHIC TABLE LASER CONTROL BOX AND TEMPERATURE CONTROL EQUIPMENT

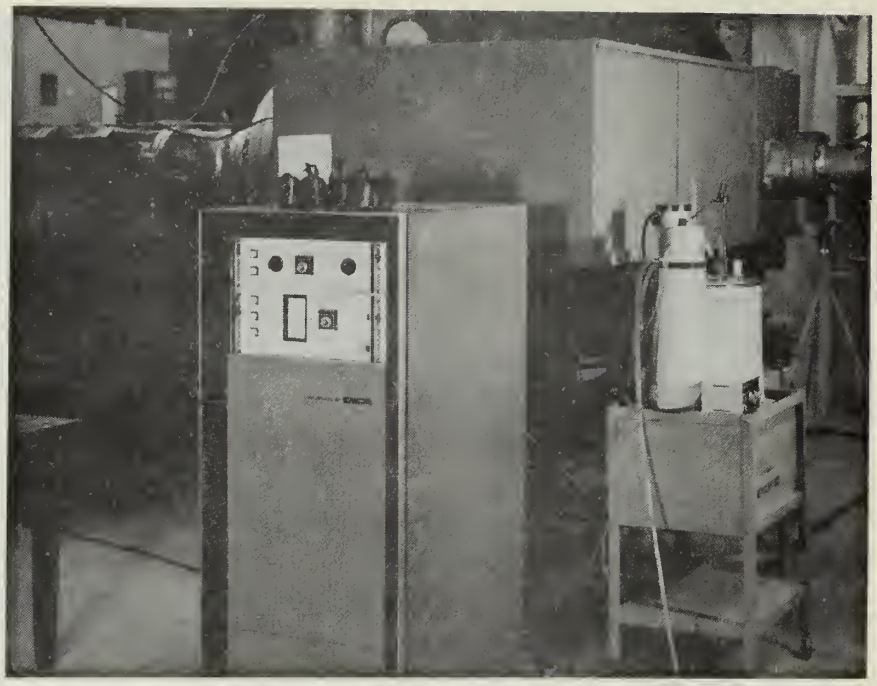


FIGURE 5b. DETAILS OF THE HOLOGRAPHIC ARRANGEMENT

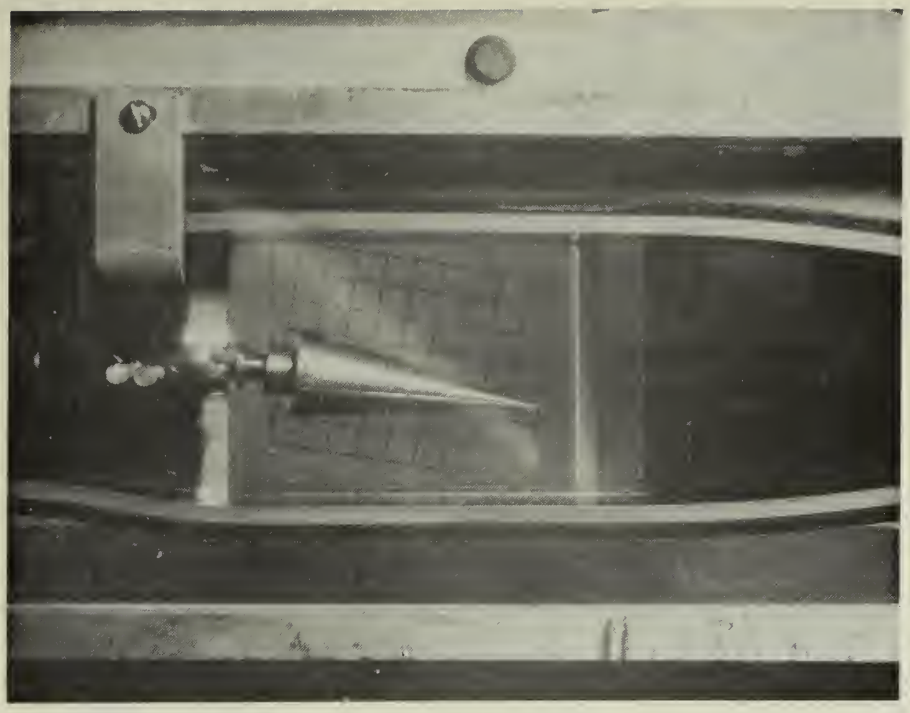


FIGURE 6. VIEW OF TEST SECTION SHOWING DETAILS OF MODEL, MOUNT, AND GRID

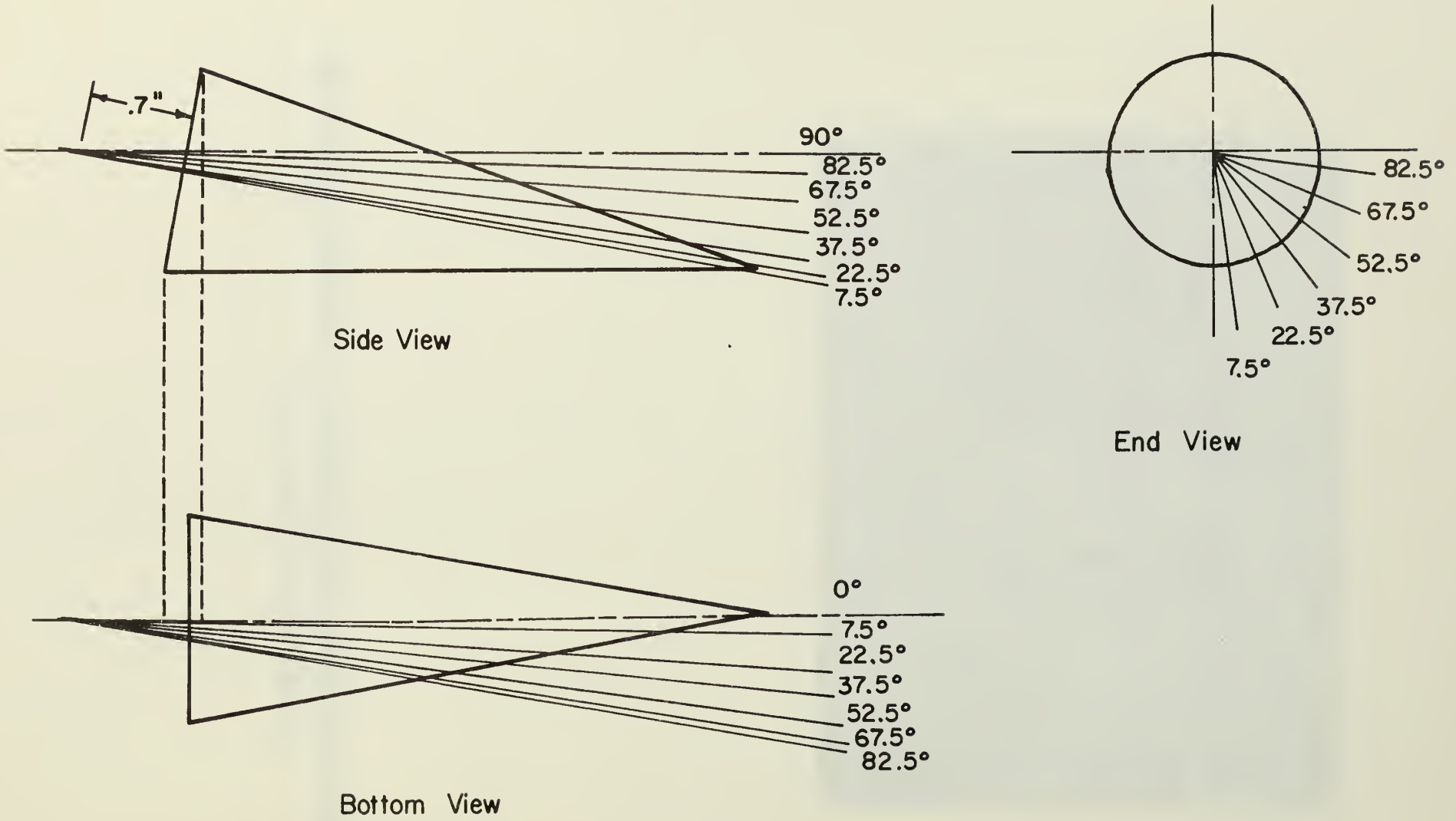
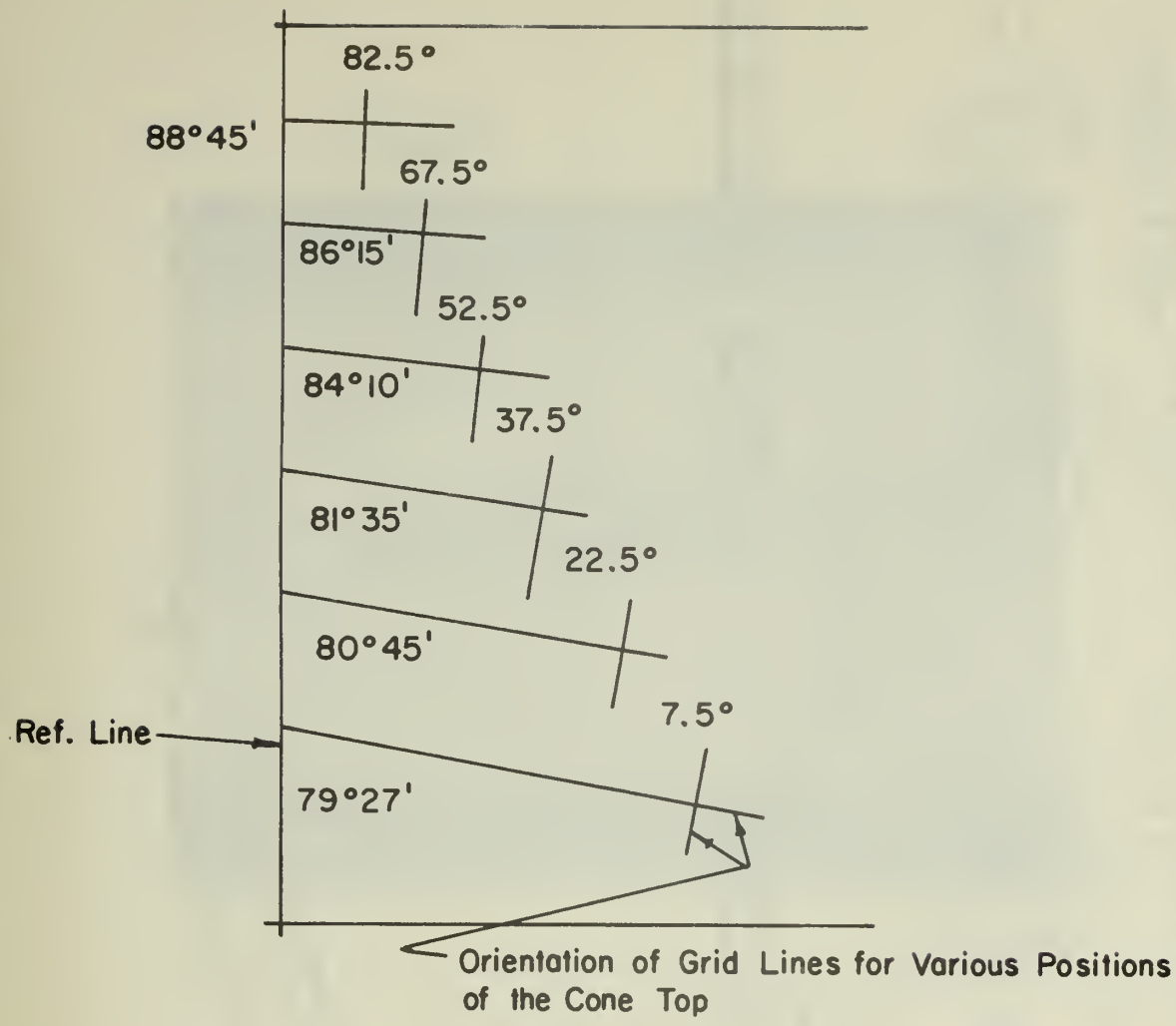


FIGURE 7a. POSITIONS OF THE CONE AXIS FOR VARIOUS ROTATIONAL POSITIONS OF THE CONE TIP



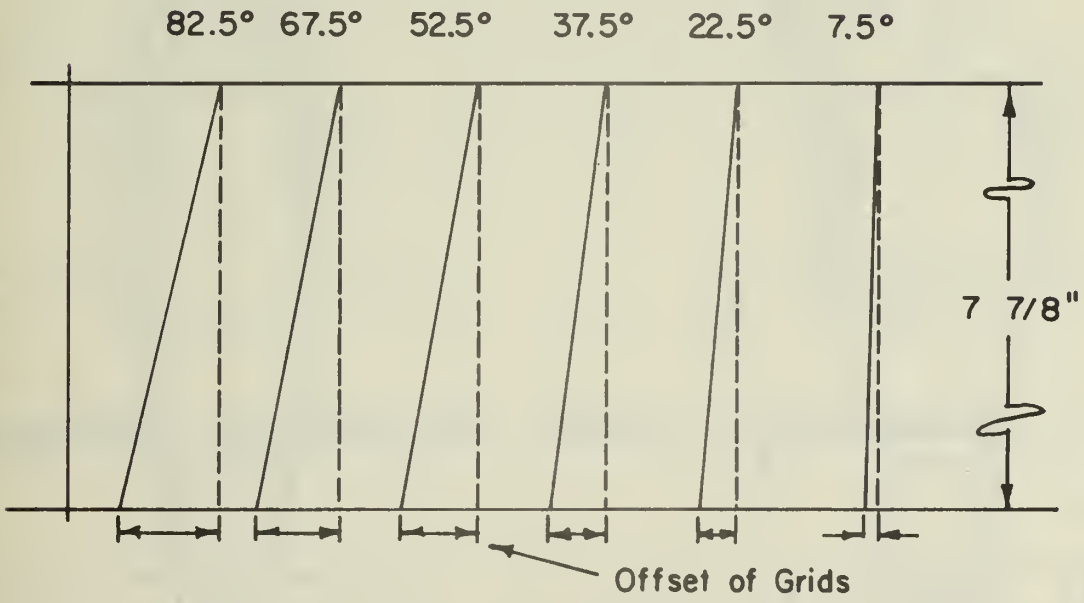
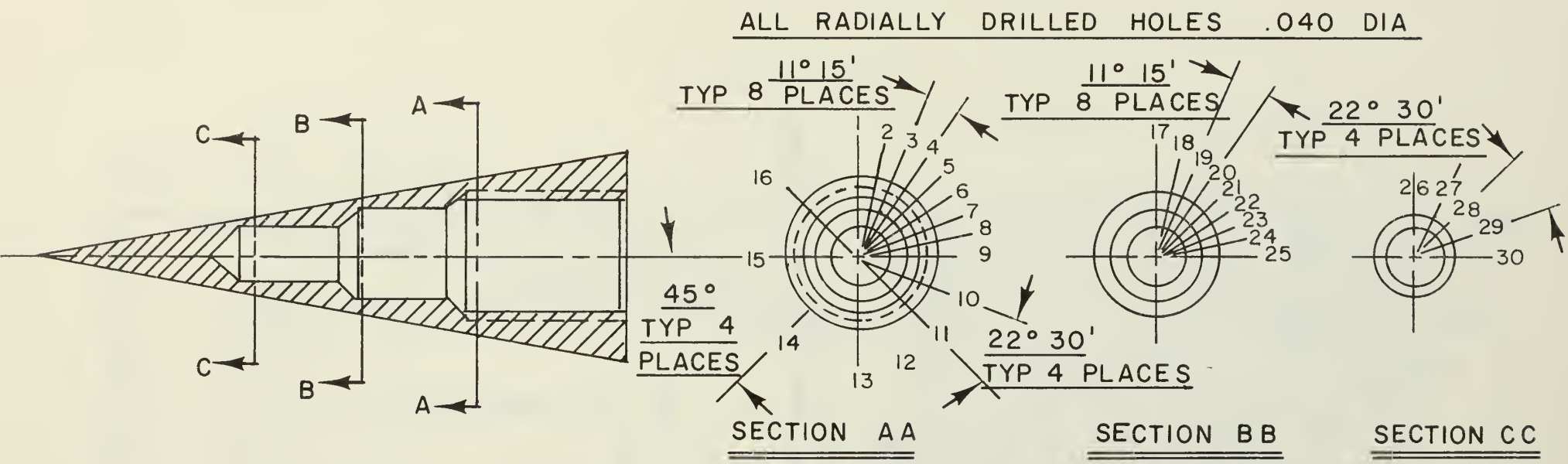
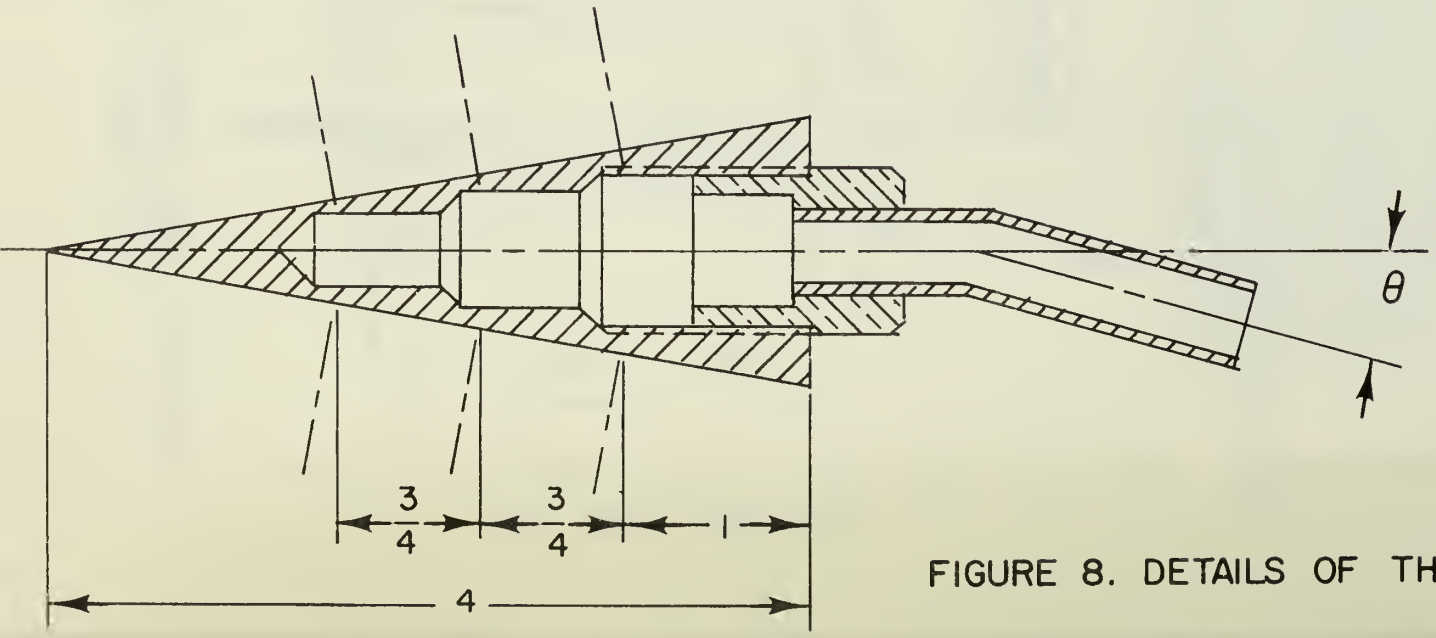


FIGURE 7b. ORIENTATION OF GRID LINES AND OFFSET OF THE GRIDS





SPINDLE	θ°
1	0
2	10

FIGURE 8. DETAILS OF THE CONE MODEL

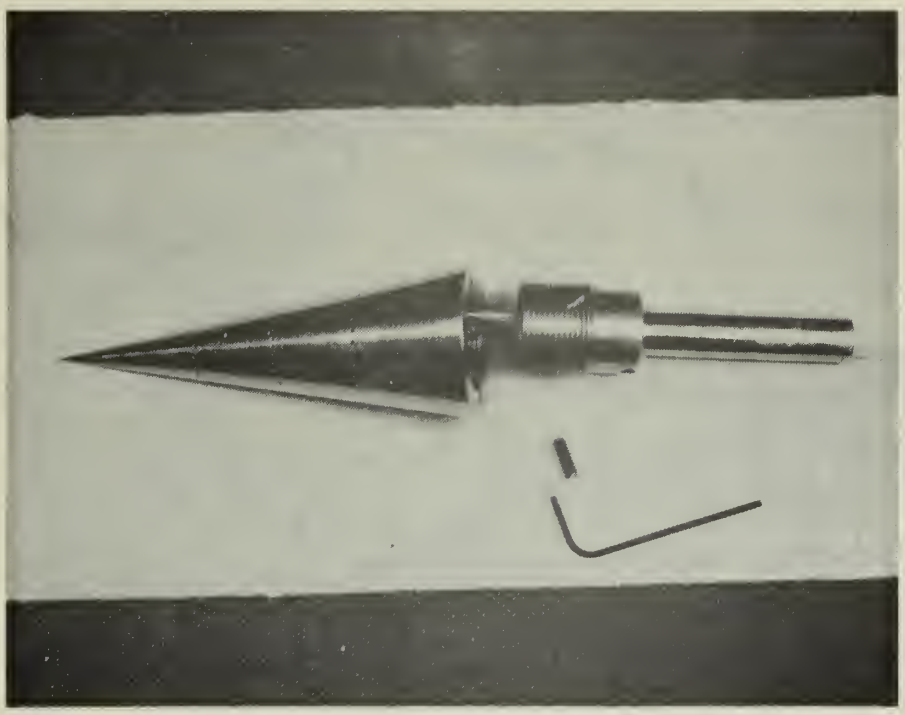


FIGURE 9. THE CONE AND SPINDLE ARRANGEMENT

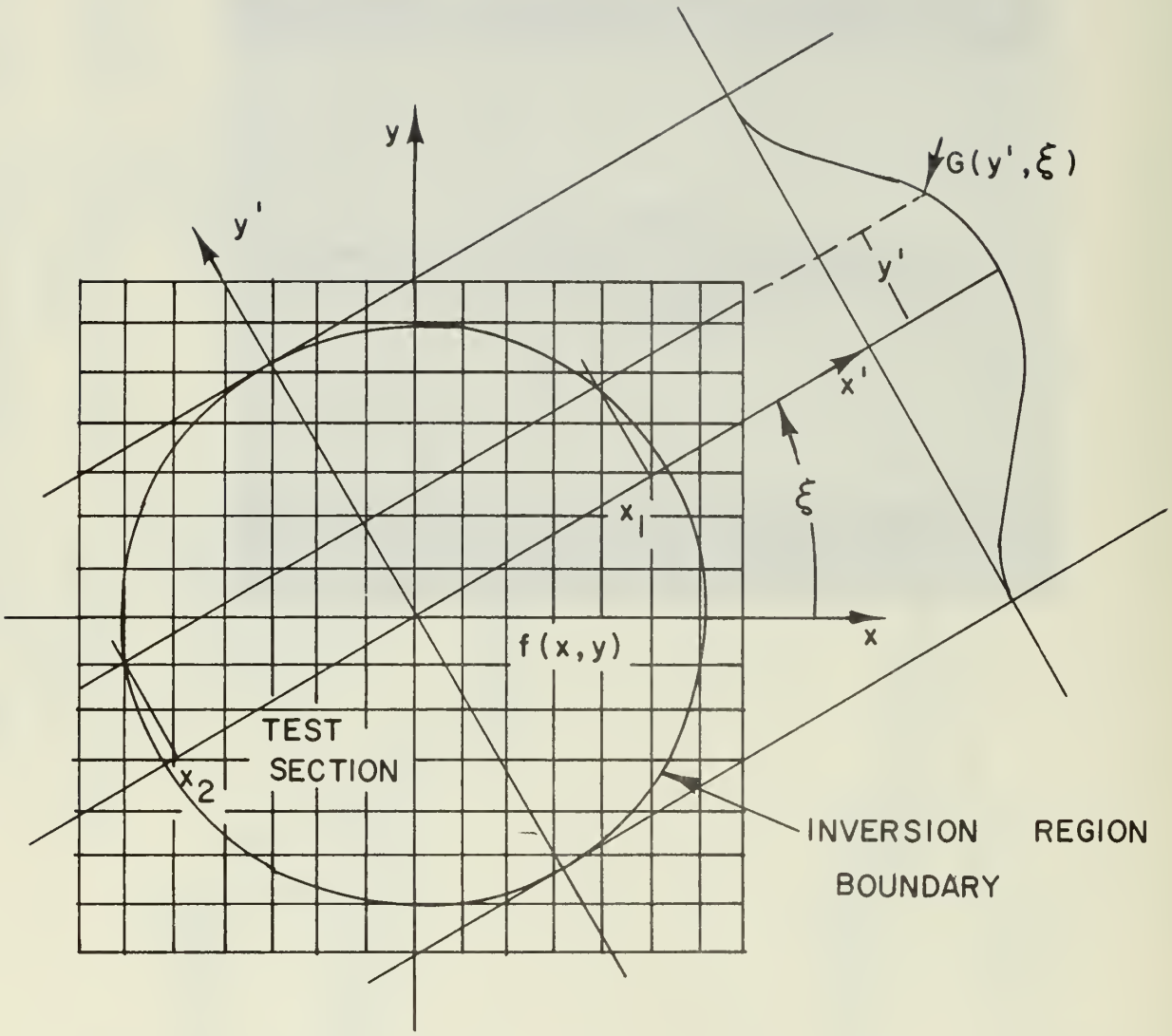


FIGURE 10. CO-ORDINATE SYSTEM USED FOR THE INVERSION OF FRINGE NUMBER TO DENSITY

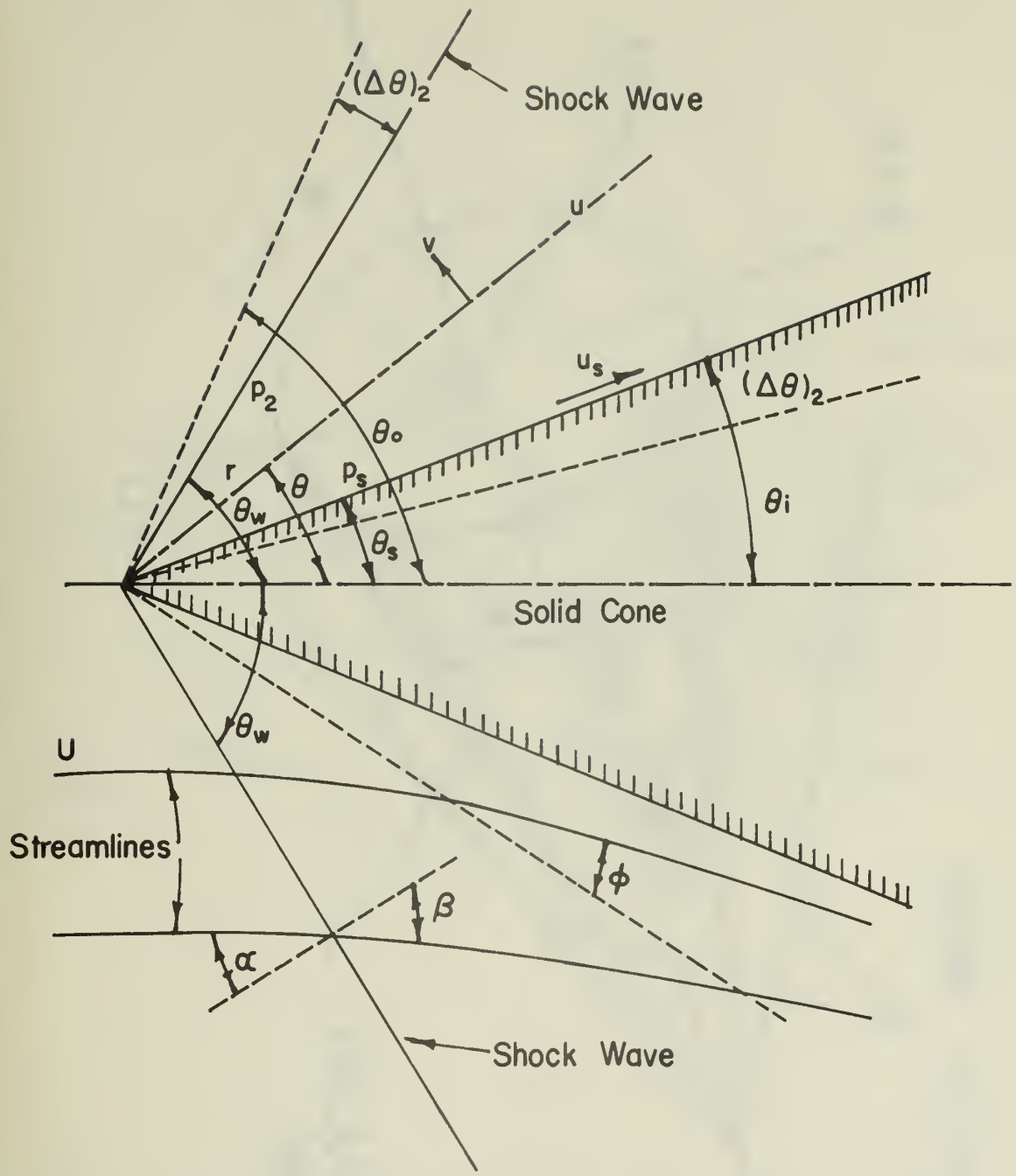


FIGURE II. CO-ORDINATE SYSTEM USED FOR THE GAS DYNAMIC EQUATION

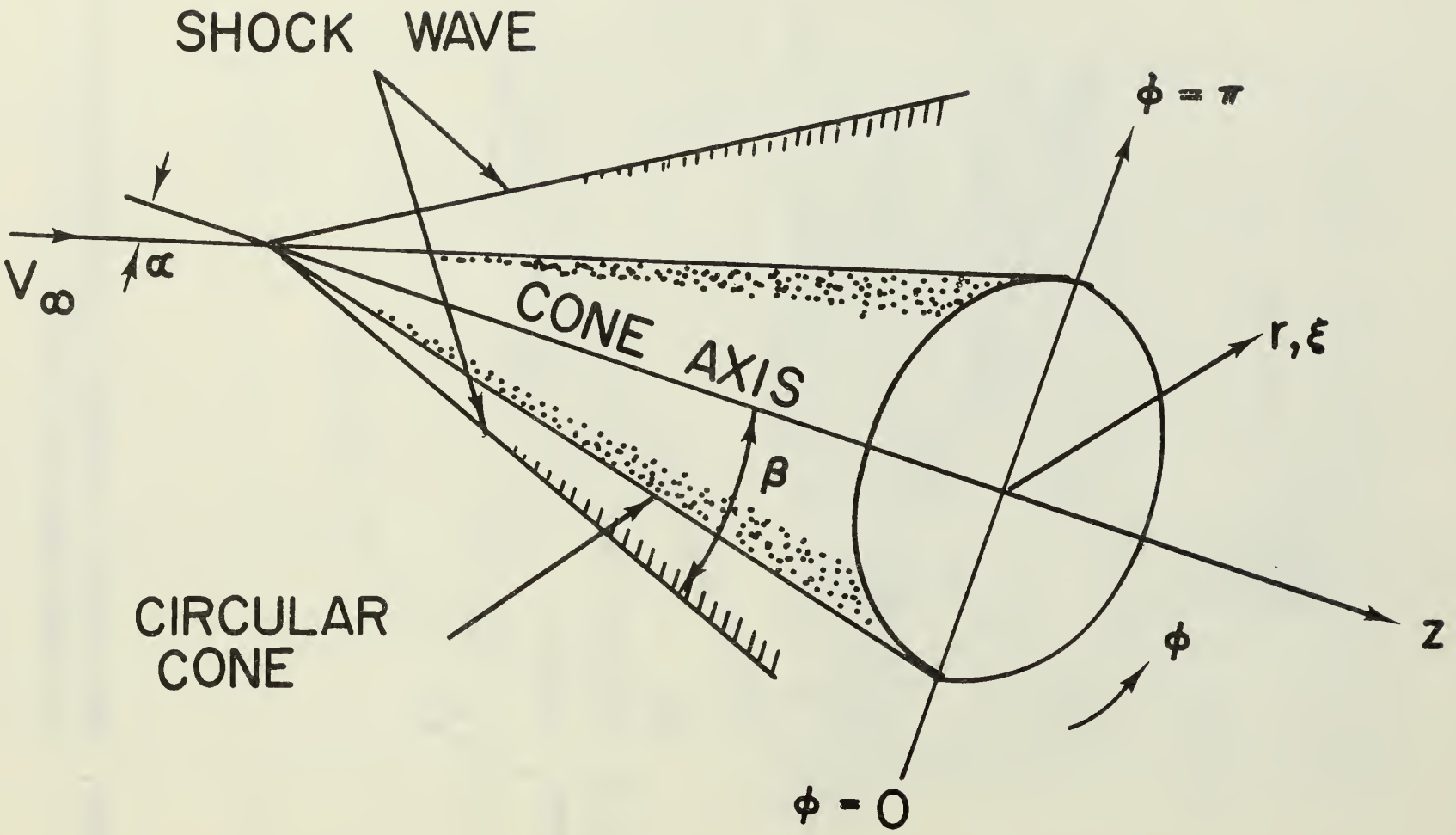


FIGURE 12. CO-ORDINATE SYSTEM USED IN REFERENCE 6

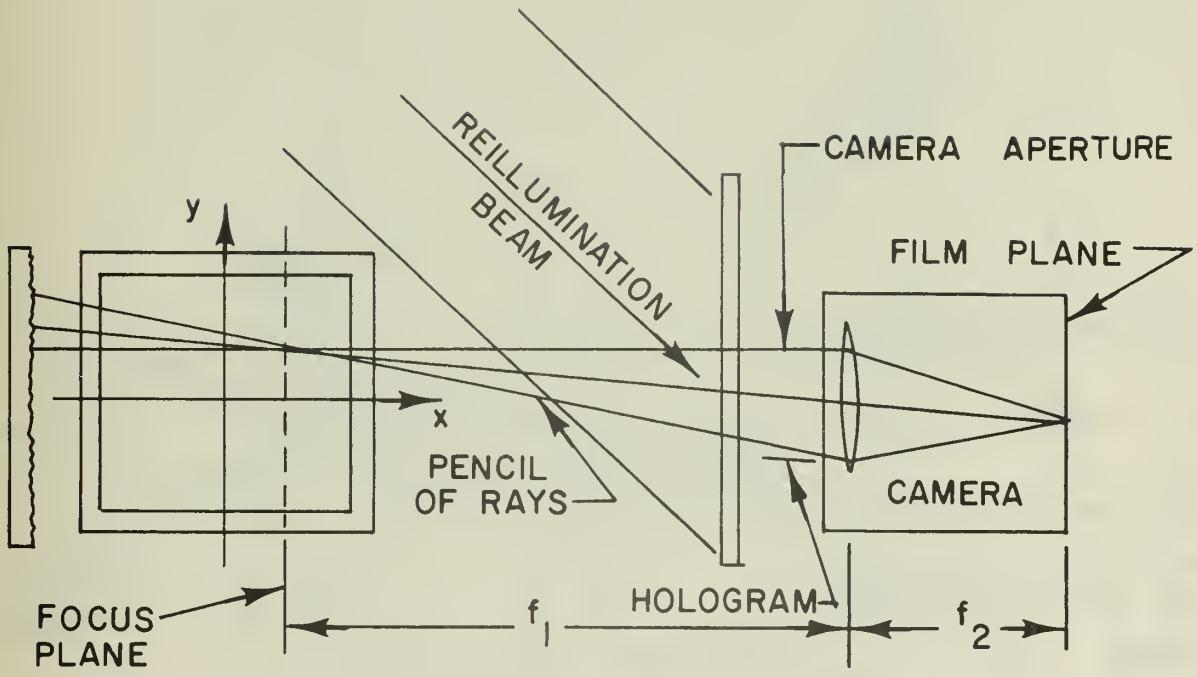


FIGURE 13. EFFECT OF APERTURE SIZE FOCUS PLANE POSITION ON THE PENCIL SIZE OF RAYS ABOUT A LINE OF SIGHT RECORDED BY CAMERA

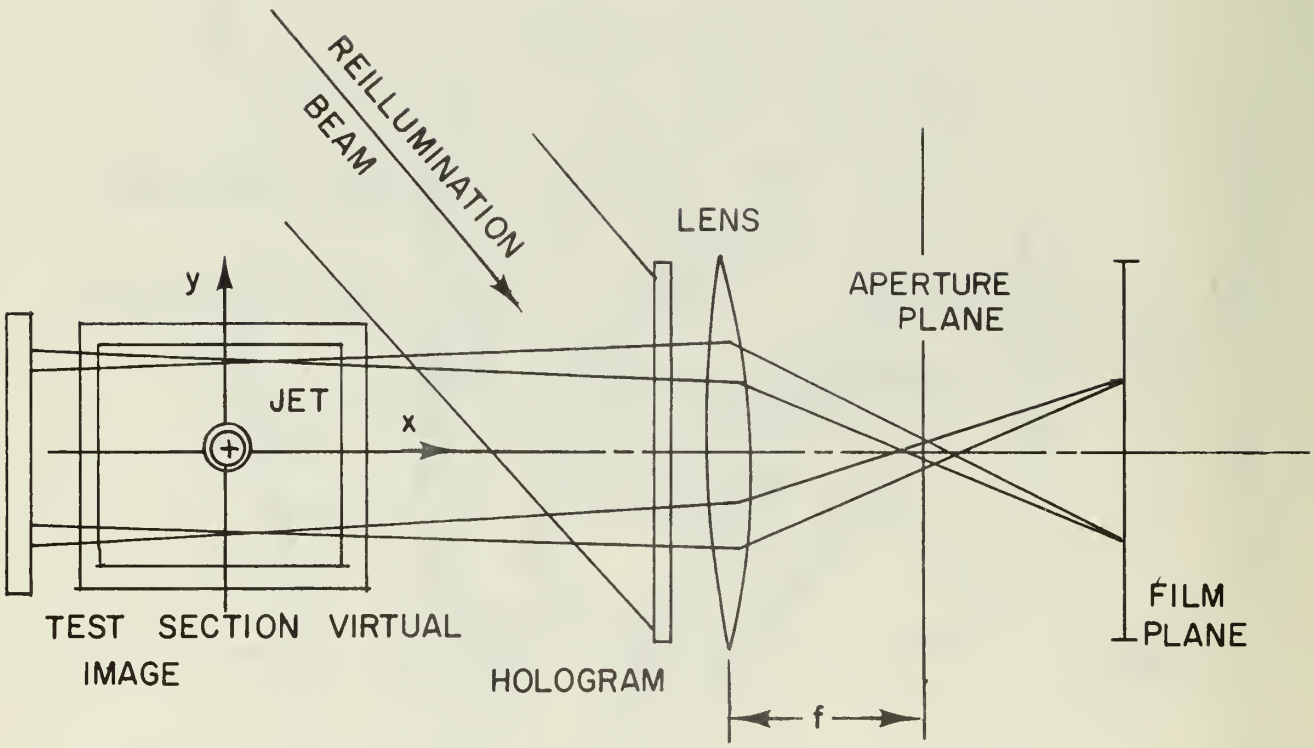


FIGURE 14. SPATIAL FILTERING TECHNIQUE FOR SELECTING PHOTOGRAPH OF CONSTANT ANGLE LINES OF LIGHT

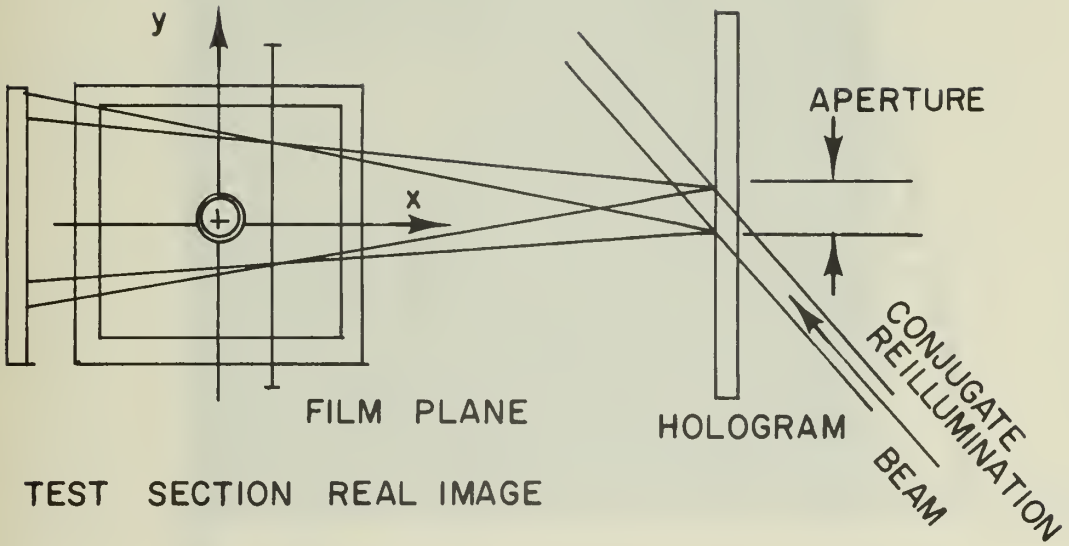


FIGURE 15. LENSLESS PHOTOGRAPHIC TECHNIQUE USING A CONJUGATE REFERENCE BEAM OF SMALL DIAMETER

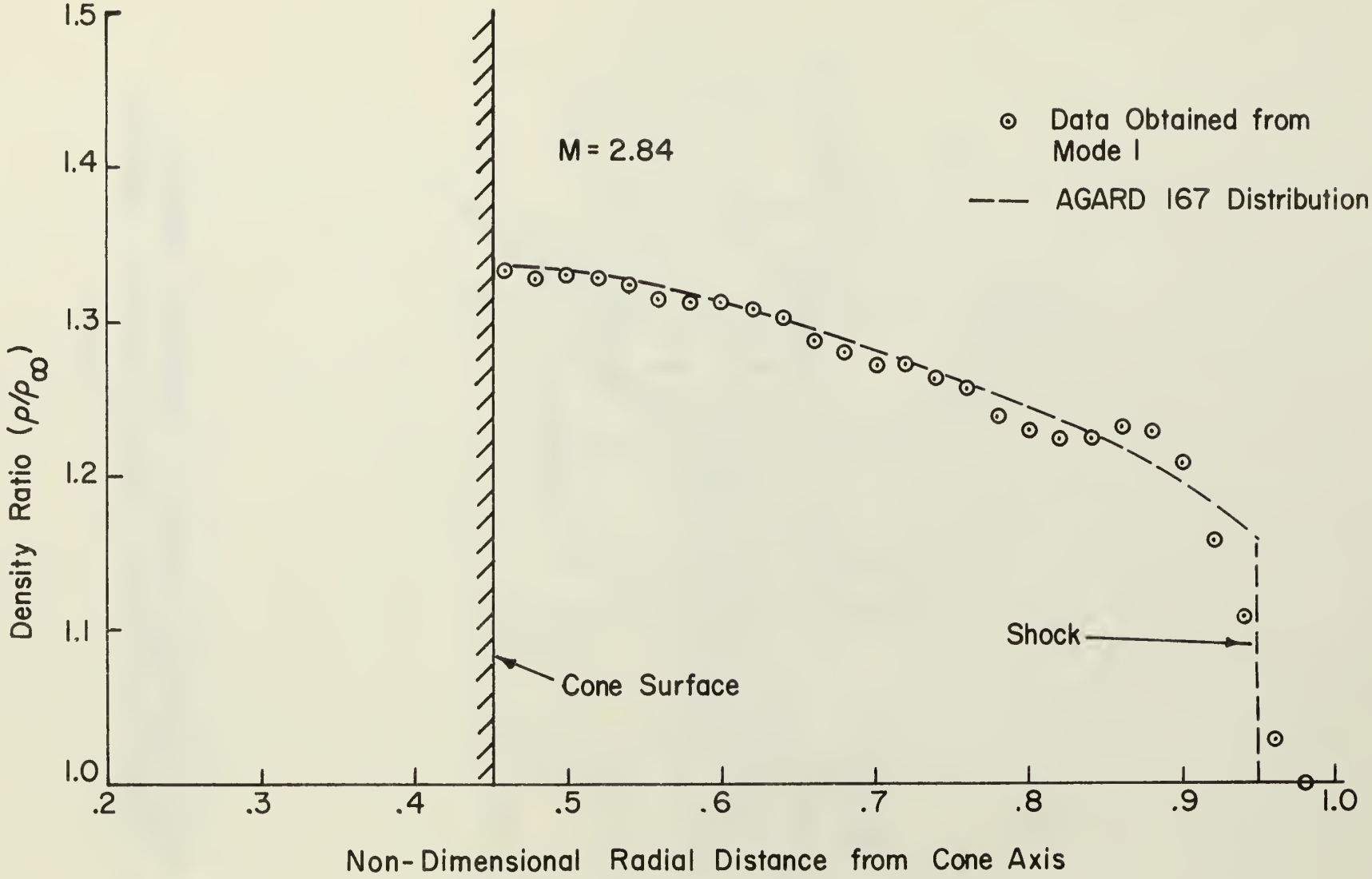


FIGURE 16
COMPARISON OF THE RADIAL DENSITY DISTRIBUTION OBTAINED FROM MODE I WITH THE AGARD 167 RADIAL DENSITY DISTRIBUTION - AXI-SYMMETRIC CASE

FIGURE 17a. HOLOGRAPHIC INTERFEROGRAM OBTAINED FOR THE AXI-SYMMETRIC CASE

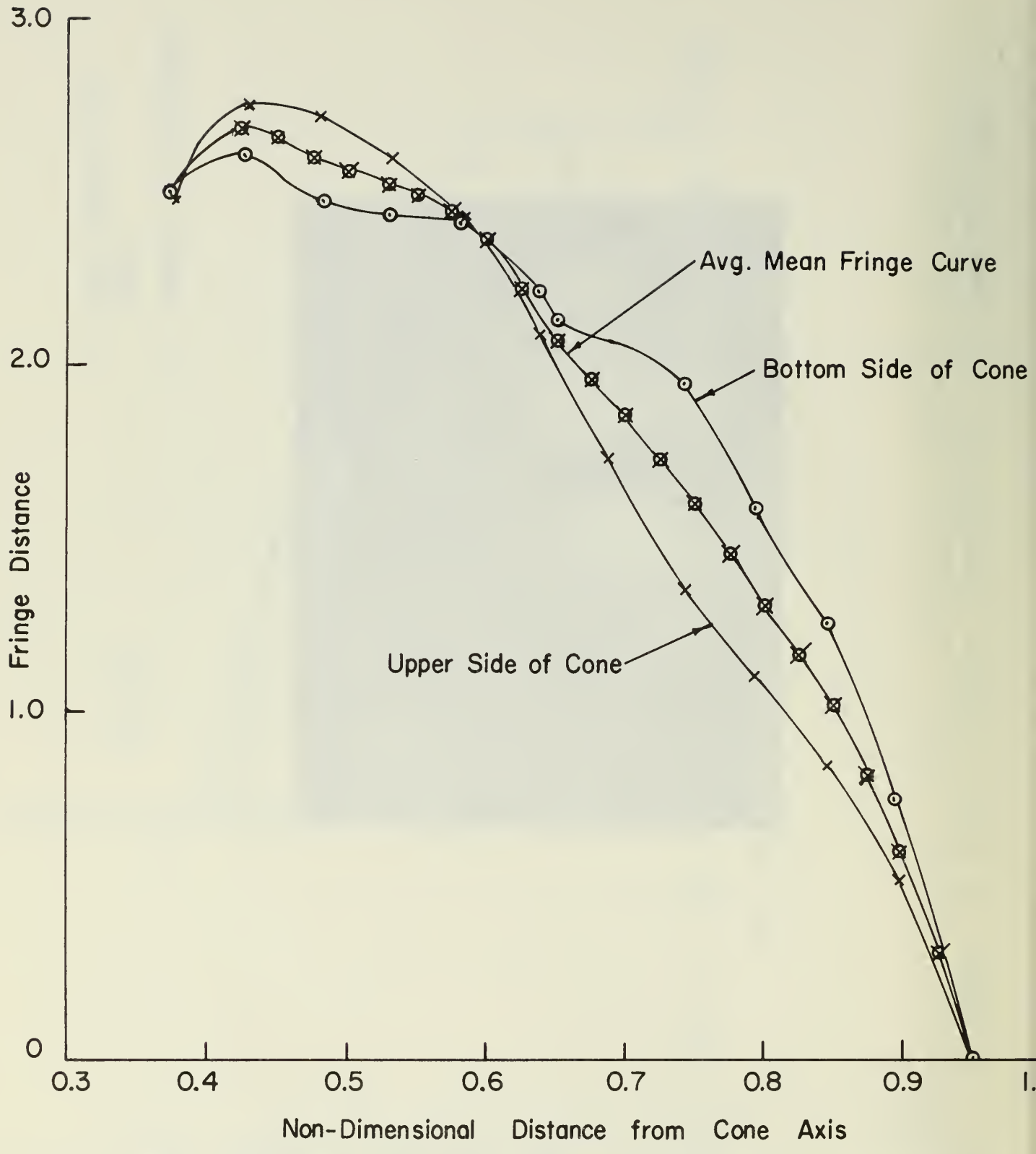


FIGURE 17b. RADIAL FRINGE DISTRIBUTION FROM A REDUCTION OF THE HOLOGRAPHIC INTERFEROGRAM FOR THE AXI-SYMMETRIC CASE

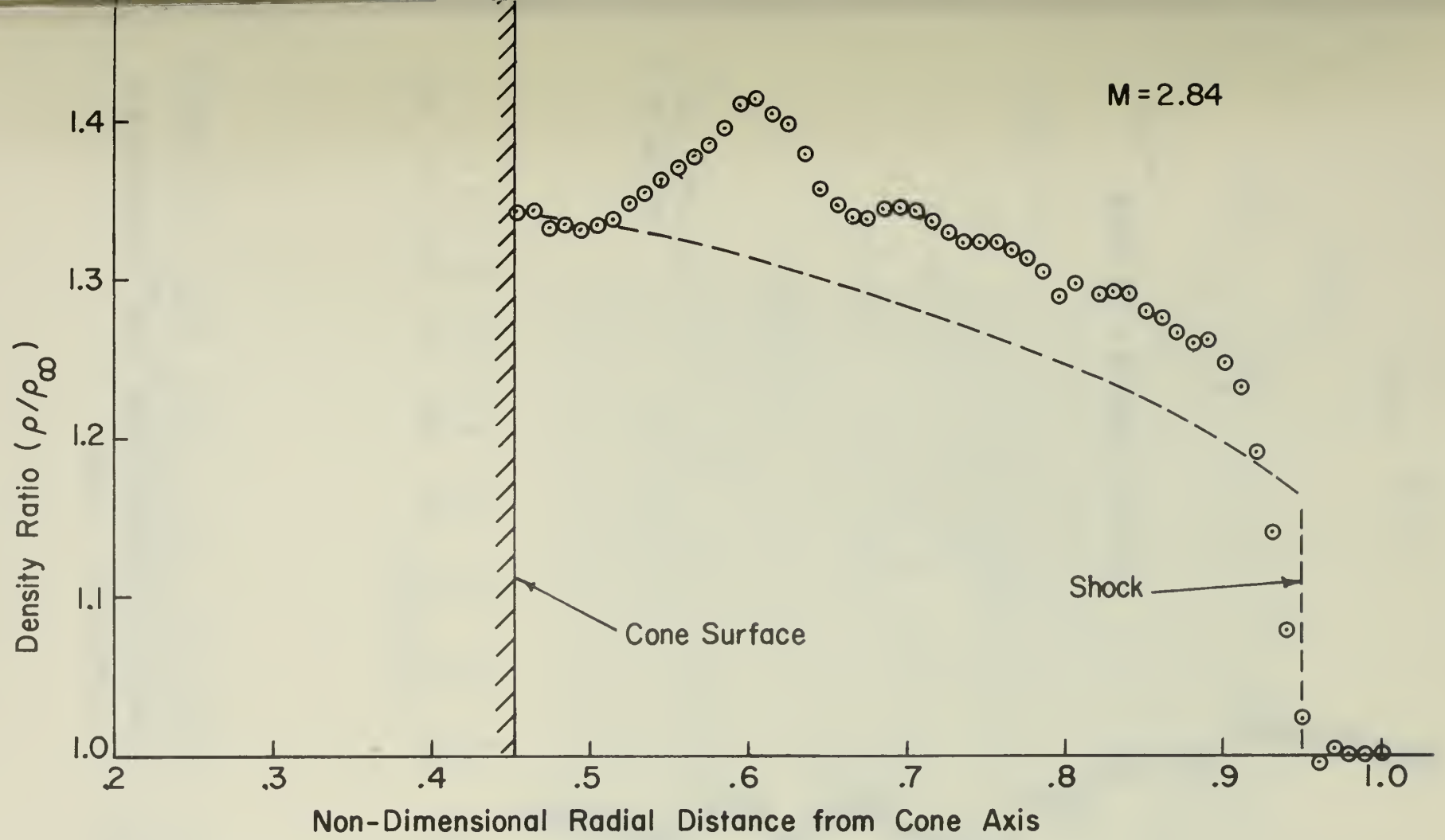


FIGURE 18.
COMPARISON OF THE RADIAL DENSITY DISTRIBUTION OBTAINED FROM EXPERIMENT WITH THE AGARD 167 DISTRIBUTION - AXI-SYMMETRIC CASE

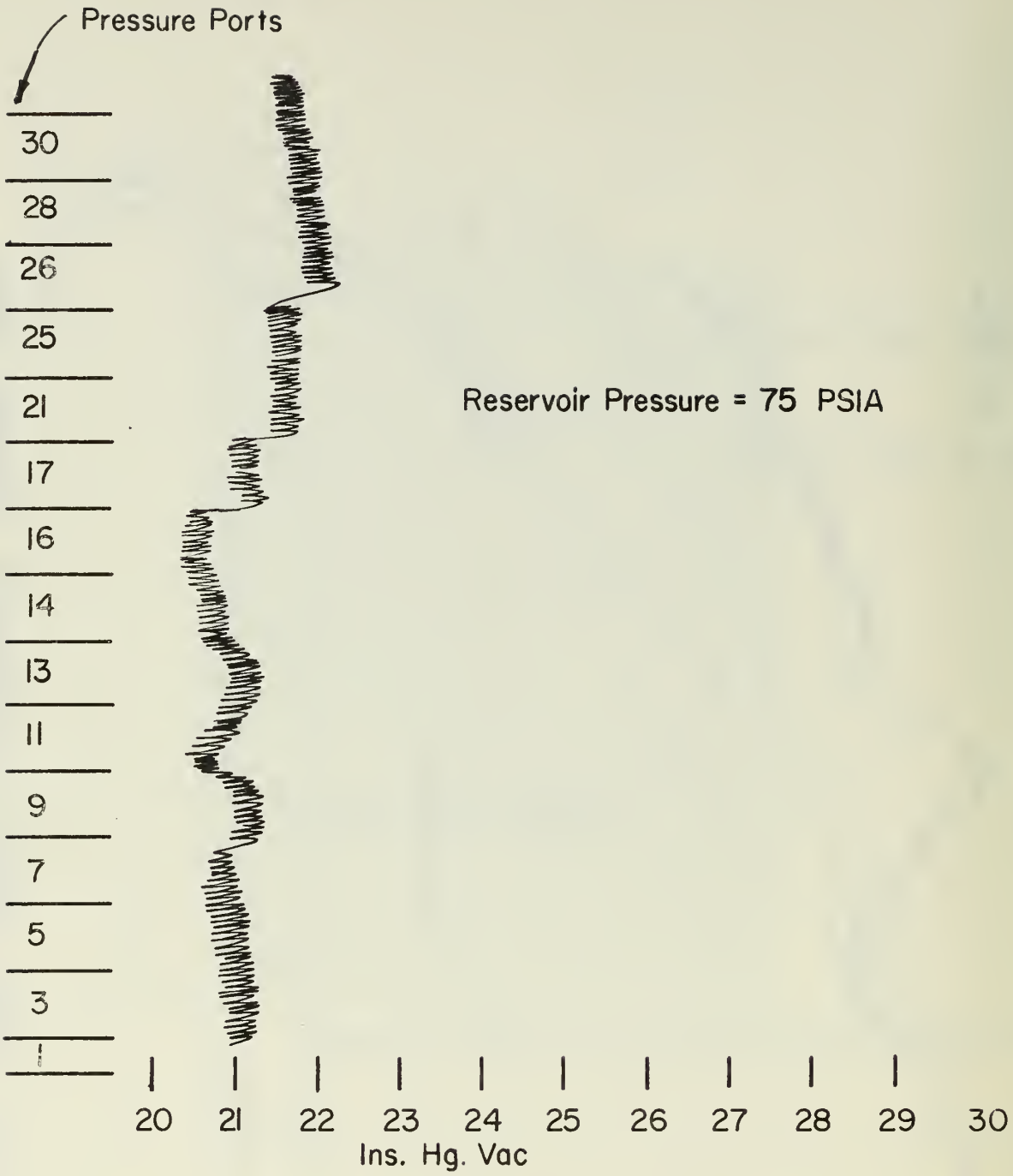


FIGURE 19. PRESSURE TRACE OBTAINED FROM THE VISCICORDER FOR THE AXI-SYMMETRIC CASE.

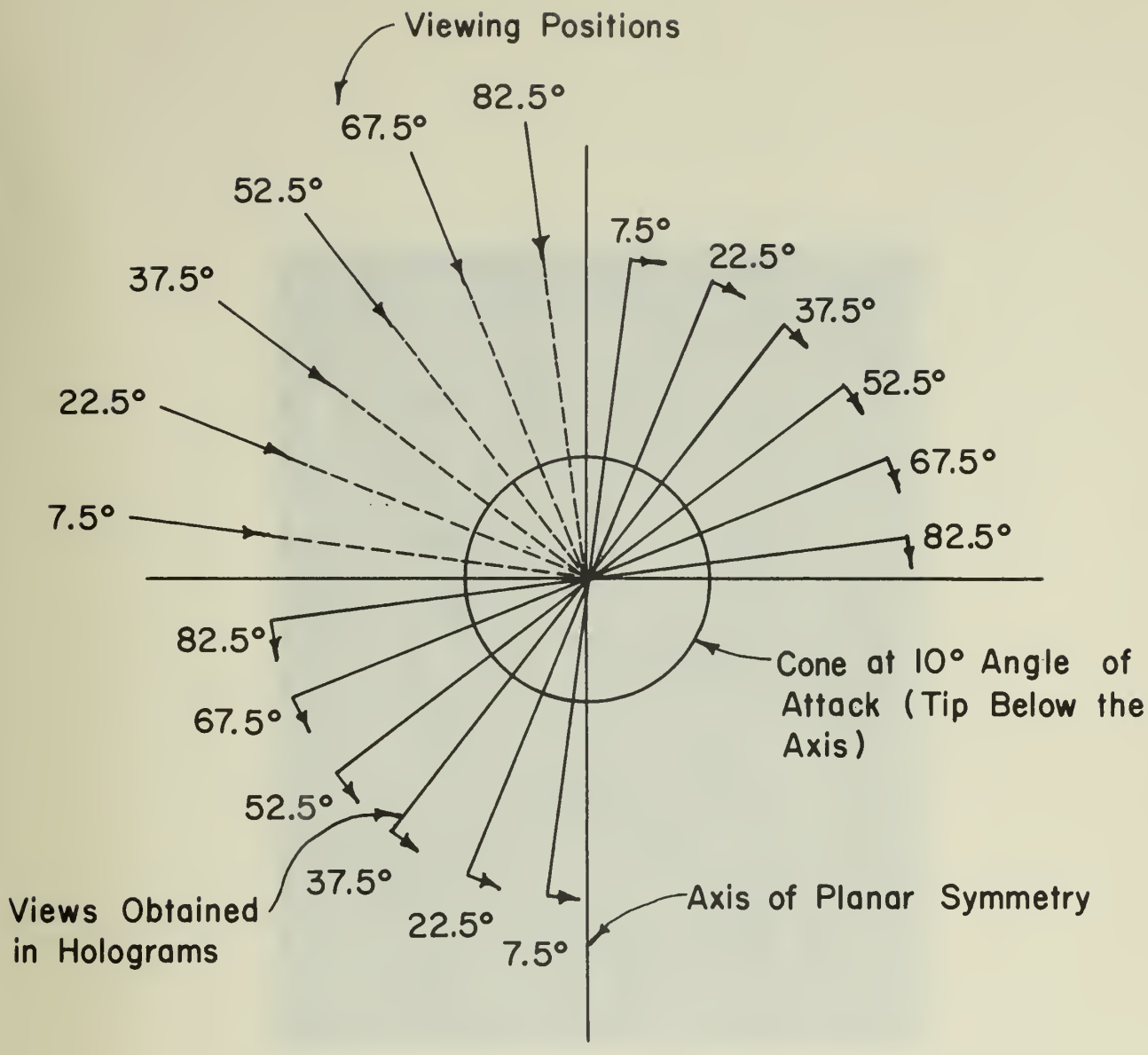


FIGURE 20. VIEWS ALONG WHICH HOLOGRAMS WERE OBTAINED FOR THE ASYMMETRIC CASE

FIGURE 21. HOLOGRAPHIC INTERFEROGRAM FOR 7.5° ANGLE OF VIEW

FIGURE 22. HOLOGRAPHIC INTERFEROGRAM FOR 22.5° ANGLE OF VIEW

FIGURE 23. HOLOGRAPHIC INTERFEROGRAM FOR 37.5° ANGLE OF VIEW

FIGURE 24. HOLOGRAPHIC INTERFEROGRAM FOR 52.5° ANGLE OF VIEW

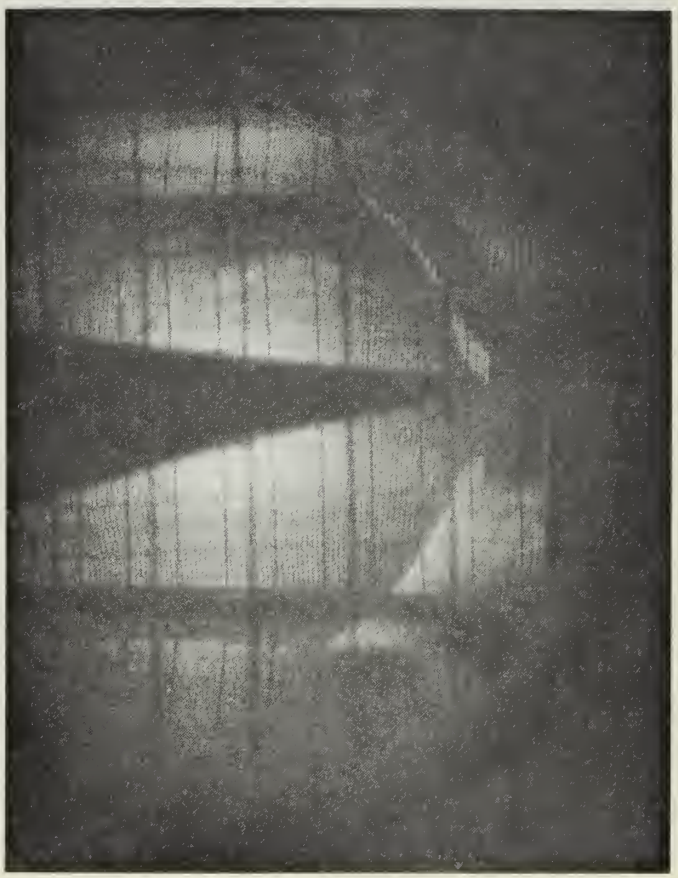


FIGURE 25. HOLOGRAPHIC INTERFEROGRAM FOR 67.5° ANGLE OF VIEW

FIGURE 26. HOLOGRAPHIC INTERFEROGRAM FOR 82.5° ANGLE OF VIEW

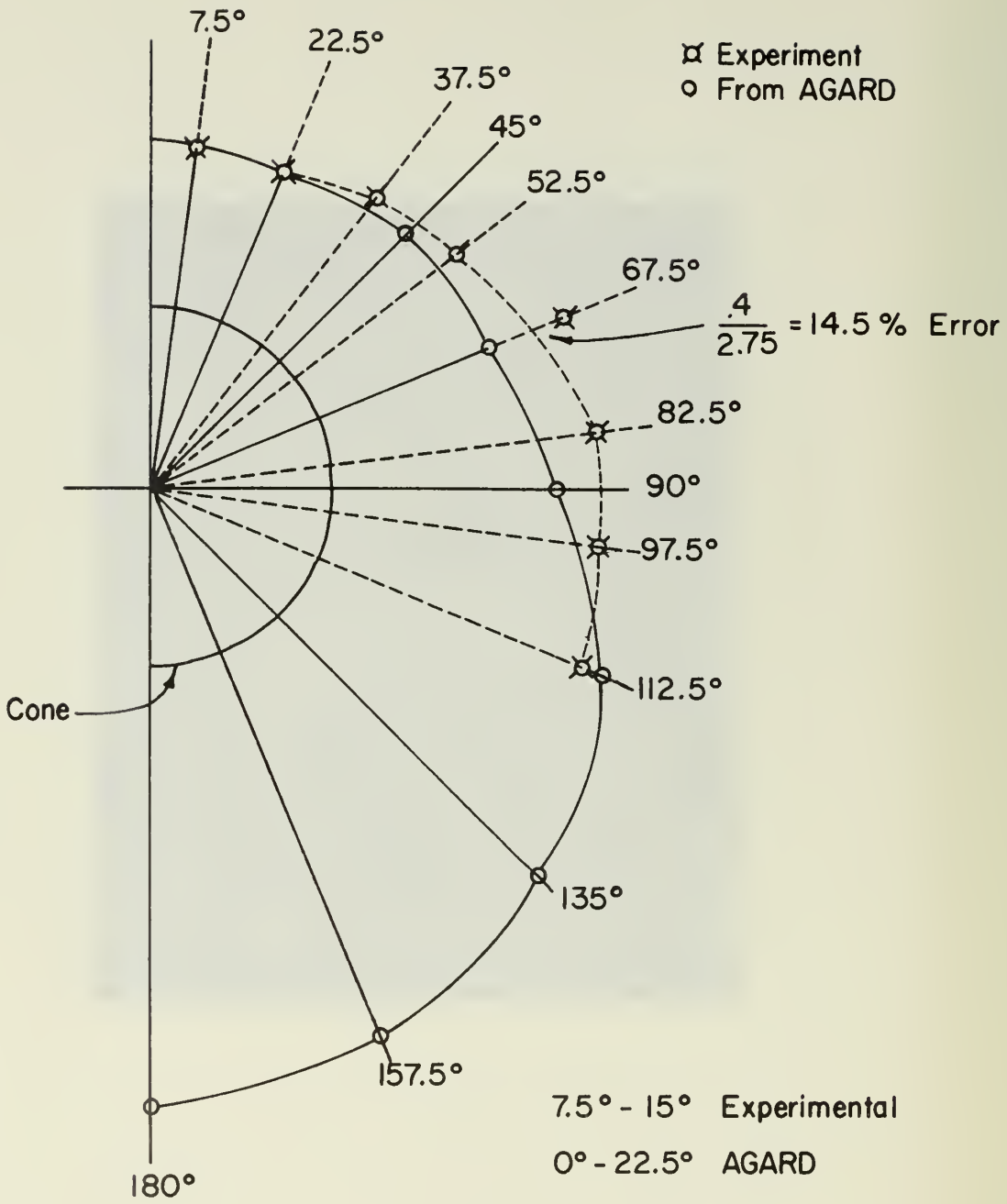


FIGURE 27. COMPARISON OF THE POSITION OF THE SHOCK WAVE OBTAINED EXPERIMENTALLY WITH THAT FROM AGARD 167 FOR M = 2.84 , ASYMMETRIC CASE

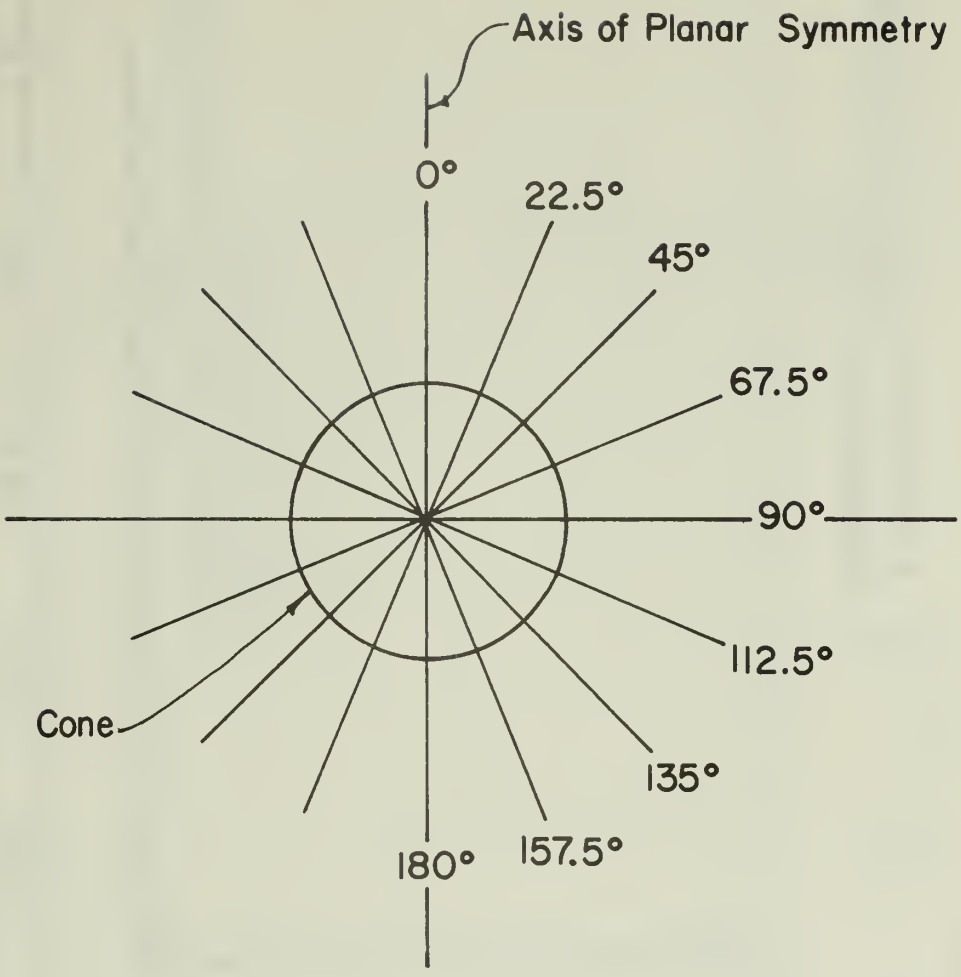


FIGURE 28. LINES OF SIGHT ALONG WHICH THE DENSITY FIELD WAS OBTAINED AFTER INVERSION, ASYMMETRIC CASE

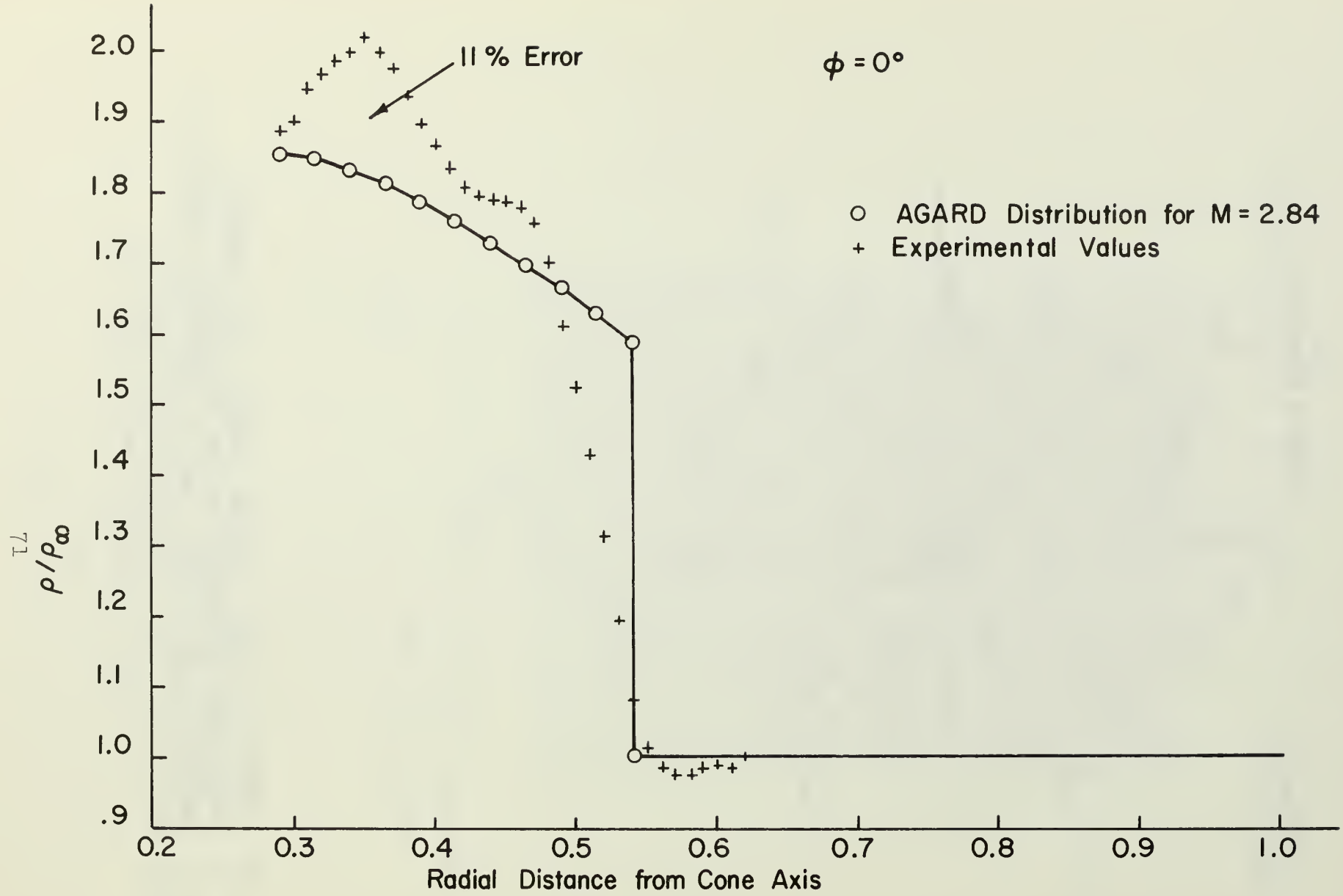


FIGURE 29. COMPARISON OF THE EXPERIMENTAL RADIAL DENSITY DISTRIBUTION WITH THE AGARD 167 DISTRIBUTION FOR THE ASYMMETRIC CASE AT M = 2.84, ϕ = 0°

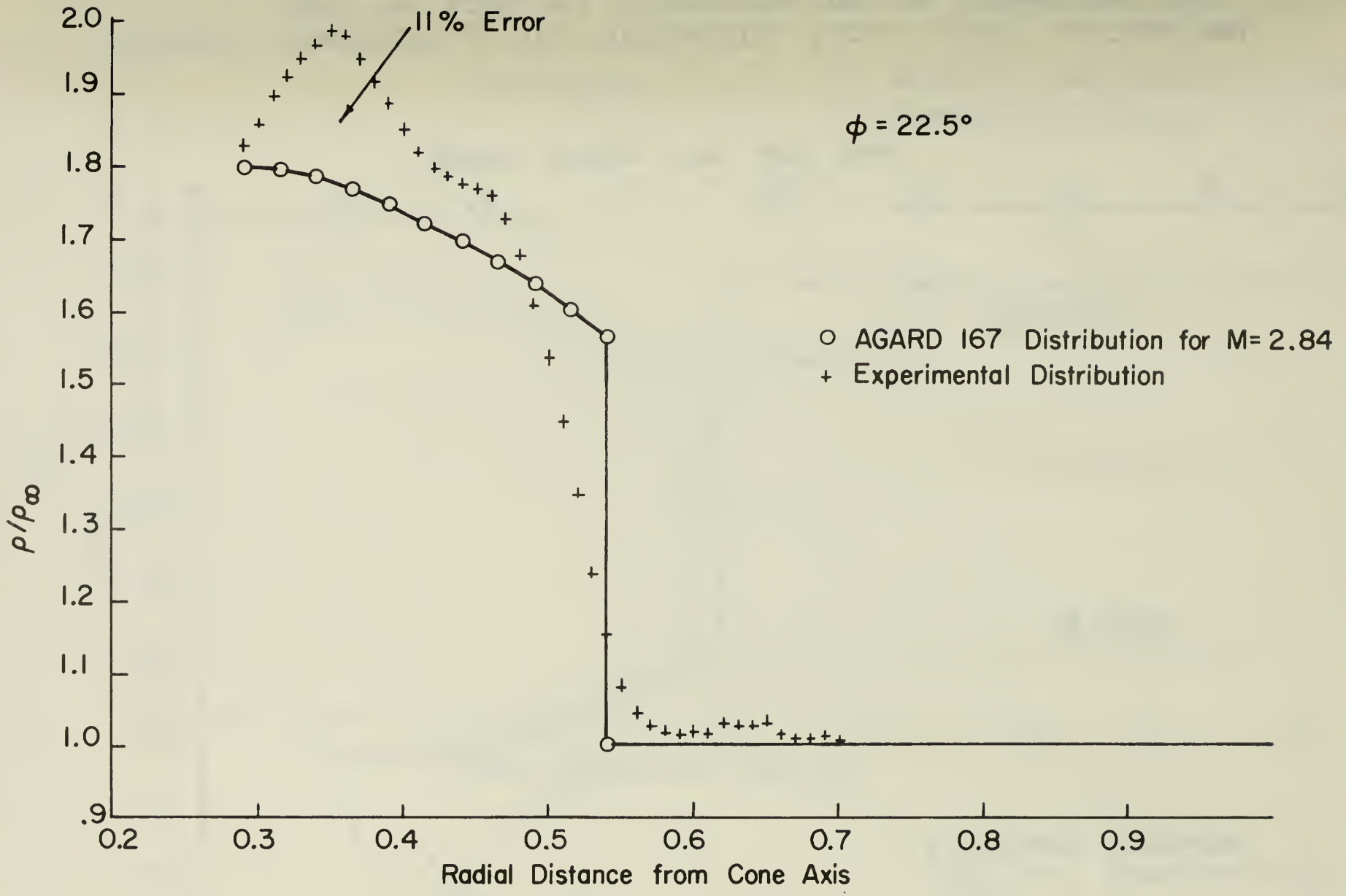


FIGURE 30. COMPARISON OF THE EXPERIMENTAL RADIAL DENSITY DISTRIBUTION WITH THE AGARD 167 DISTRIBUTION FOR THE ASYMMETRIC CASE AT M=2.84, ϕ = 22.5°

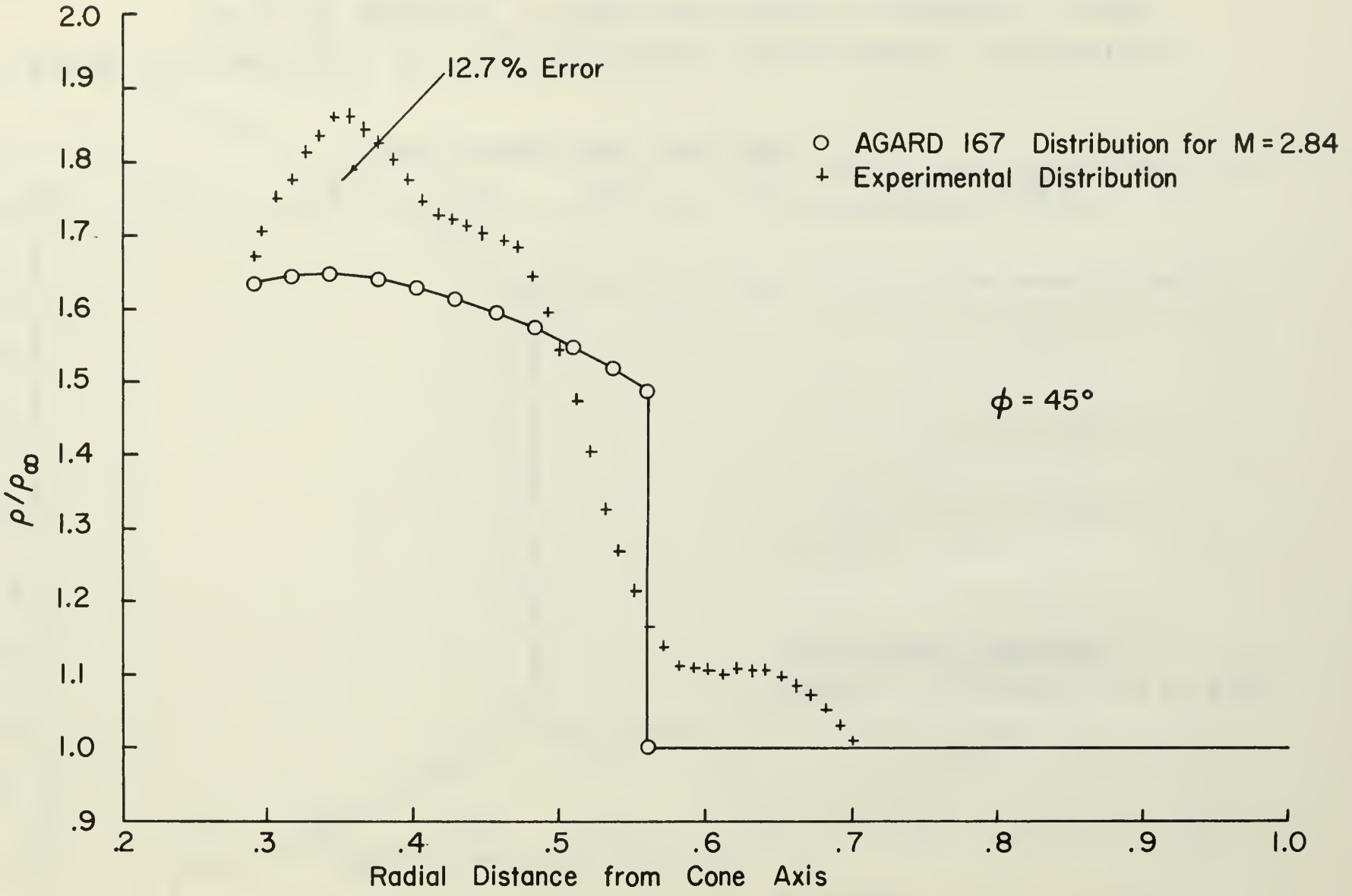


FIGURE 31. COMPARISON OF THE EXPERIMENTAL RADIAL DENSITY DISTRIBUTION WITH THE AGARD 167 DISTRIBUTION FOR THE ASYMMETRIC CASE AT M=2.84, ϕ = 45°

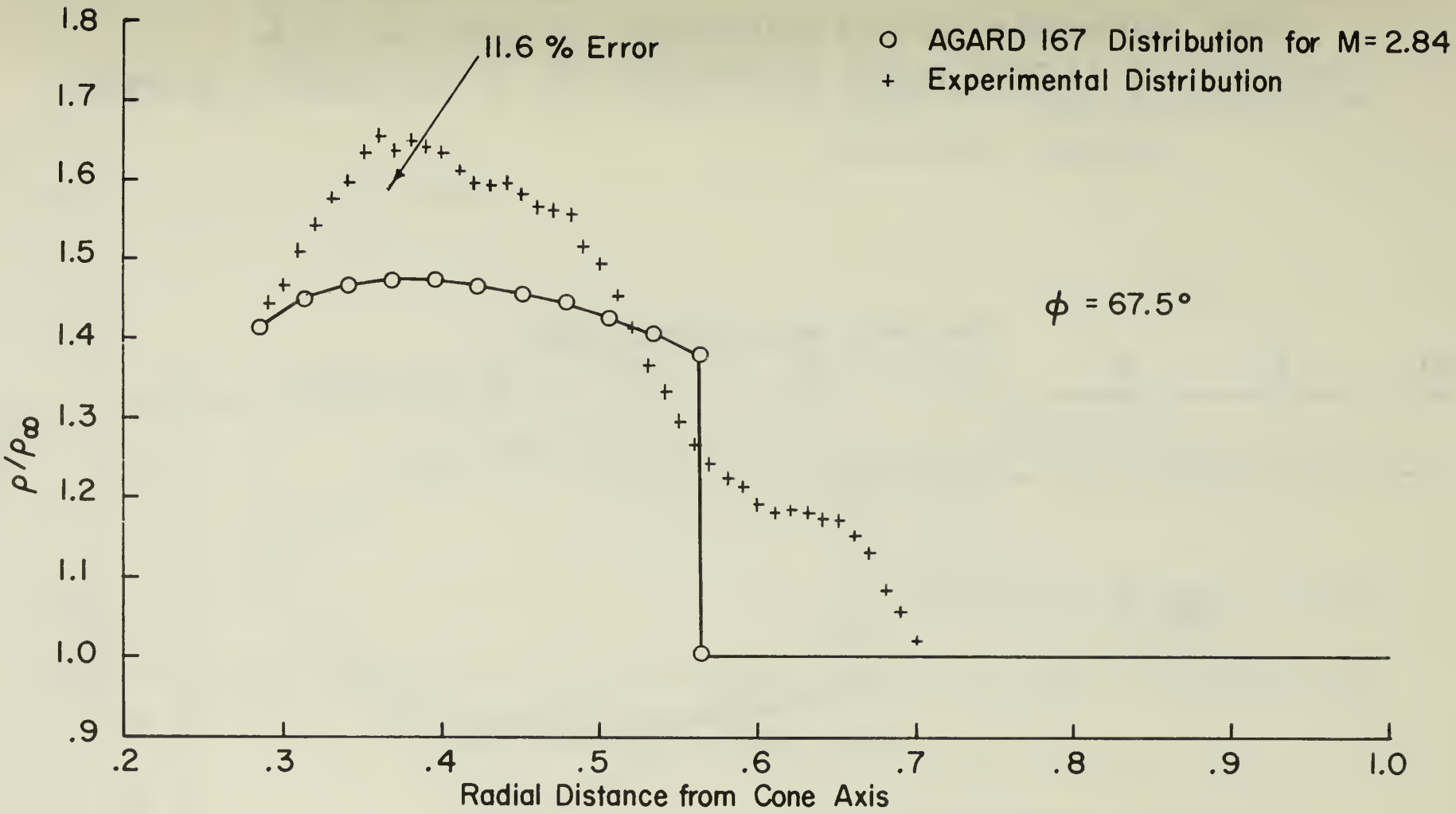


FIGURE 32. COMPARISON OF THE EXPERIMENTAL RADIAL DENSITY DISTRIBUTION WITH THE AGARD 167 DISTRIBUTION FOR THE ASYMMETRIC CASE AT M = 2.84, ϕ = 67.5°

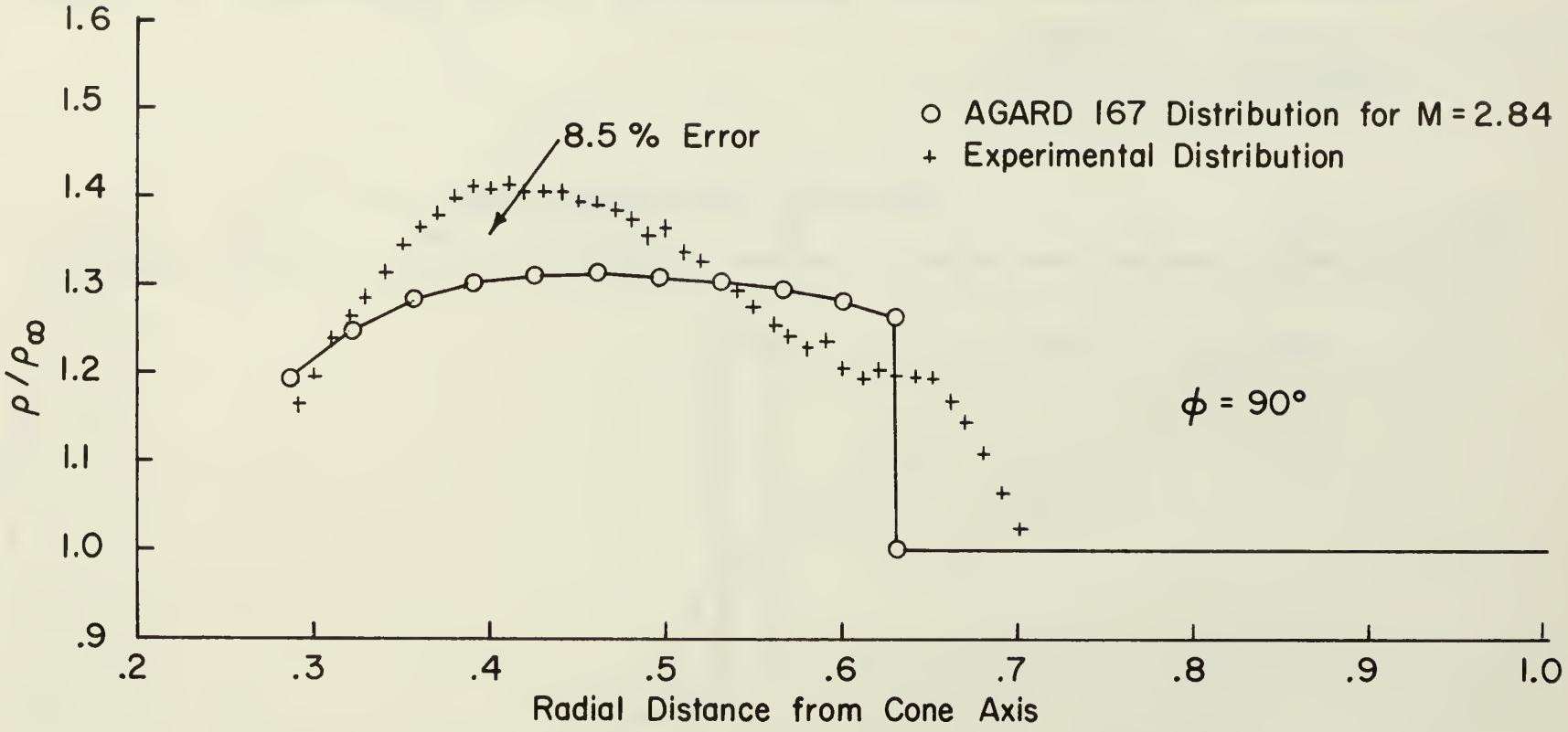


FIGURE 33. COMPARISON OF THE EXPERIMENTAL RADIAL DENSITY DISTRIBUTION WITH THE AGARD 167 DISTRIBUTION FOR THE ASYMMETRIC CASE AT M = 2.84, ϕ = 90°

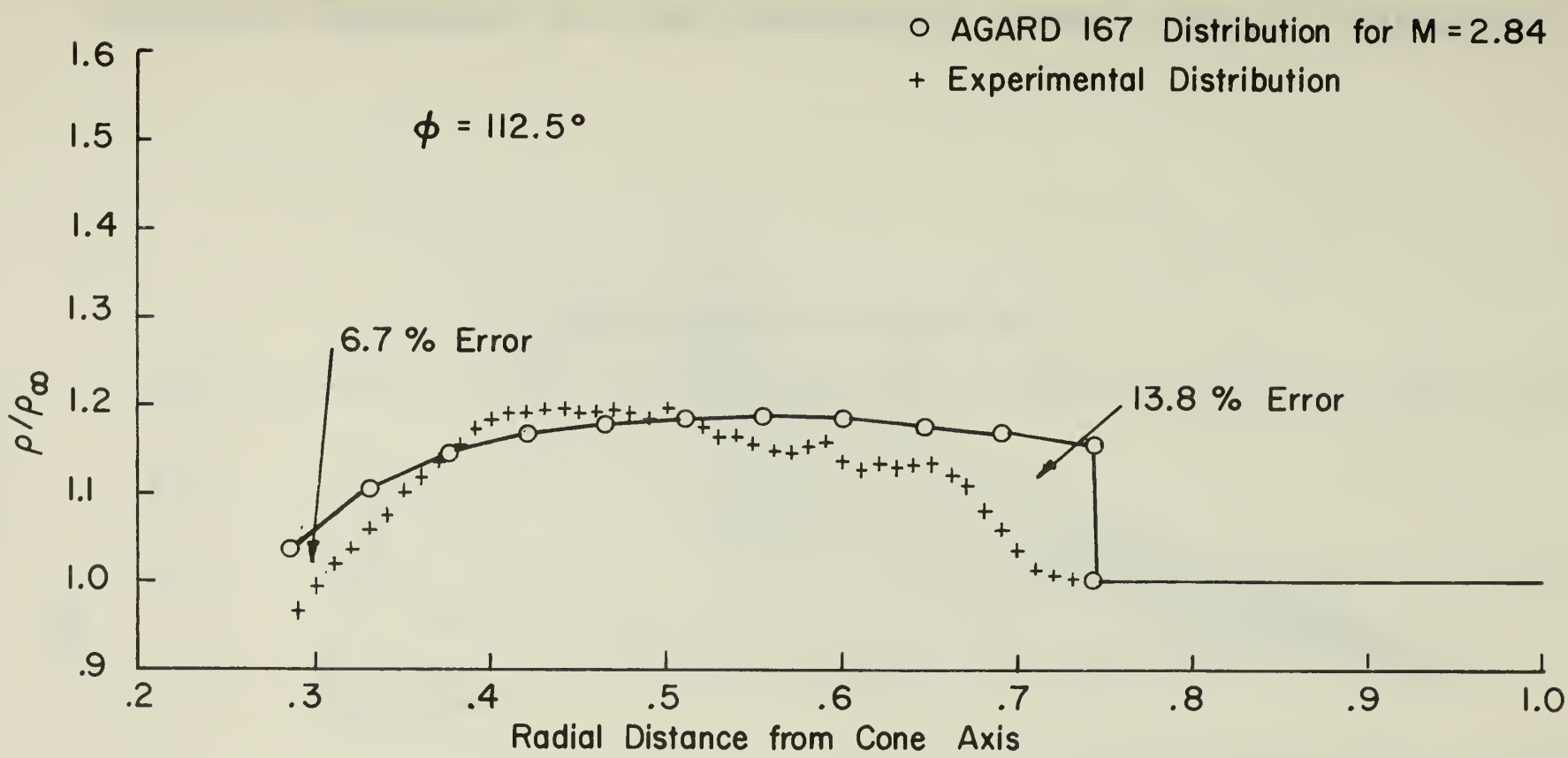


FIGURE 34. COMPARISON OF THE EXPERIMENTAL RADIAL DENSITY DISTRIBUTION WITH THE AGARD 167 DISTRIBUTION FOR THE ASYMMETRIC CASE AT M = 2.84, ϕ = 112.5°

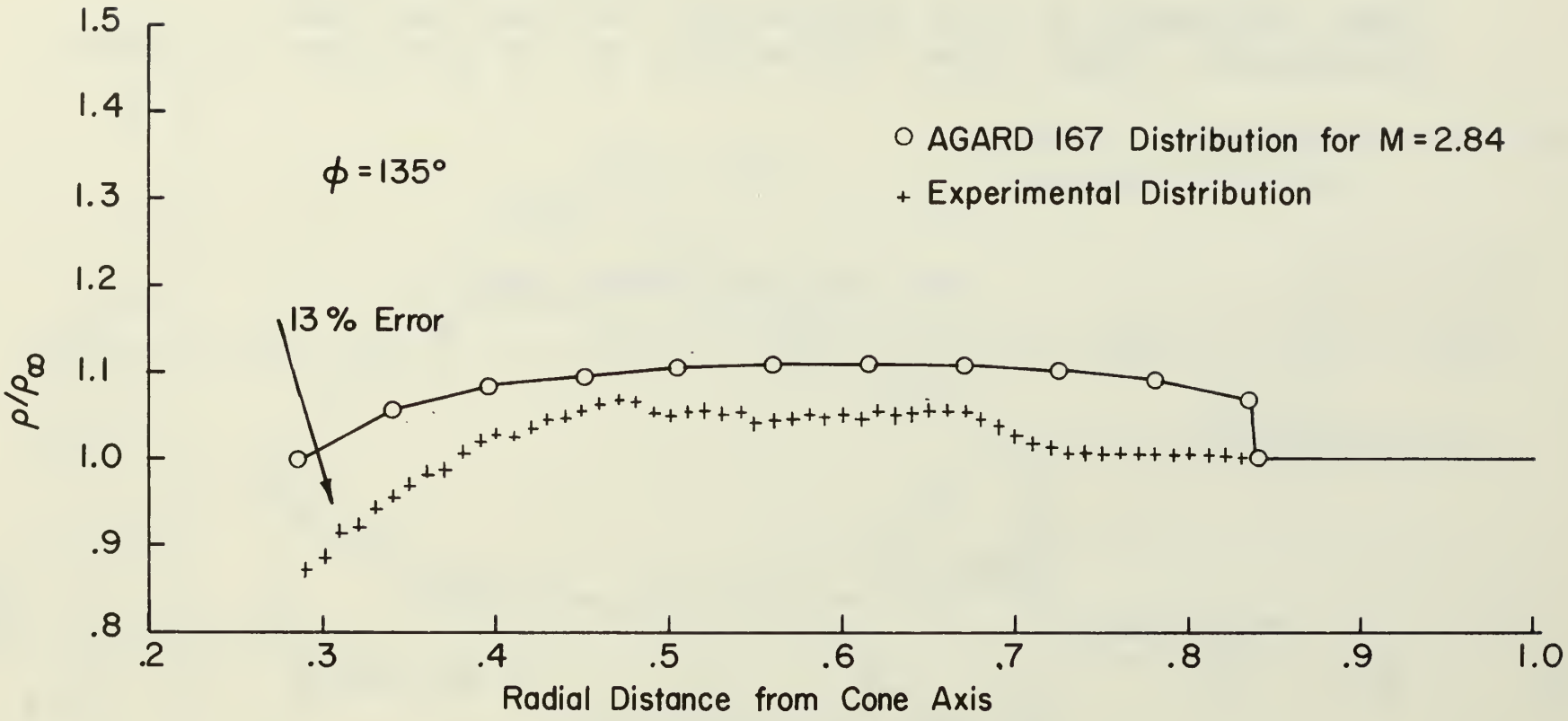
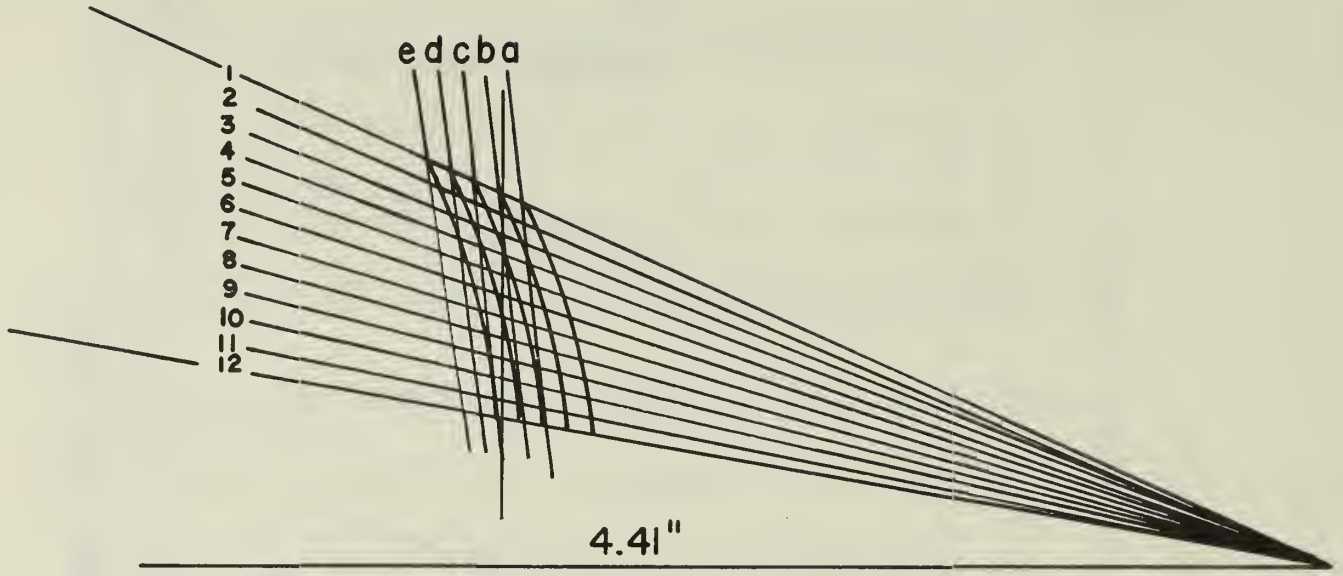


FIGURE 35. COMPARISON OF THE EXPERIMENTAL RADIAL DENSITY DISTRIBUTION WITH THE AGARD 167 DISTRIBUTION FOR THE ASYMMETRIC CASE AT M = 2.84, ϕ = 135°

Avg. Fringe Dist. = $\frac{11.1}{4}$ = 2.775



Magnif. Factor = $\frac{4.1"}{2.25"}$ = 1.8222"

FIGURE 36. TRACING OF THE PROJECTED IMAGE OF THE HOLOGRAPHIC INTERFEROGRAM FOR THE AXI-SYMMETRIC CASE

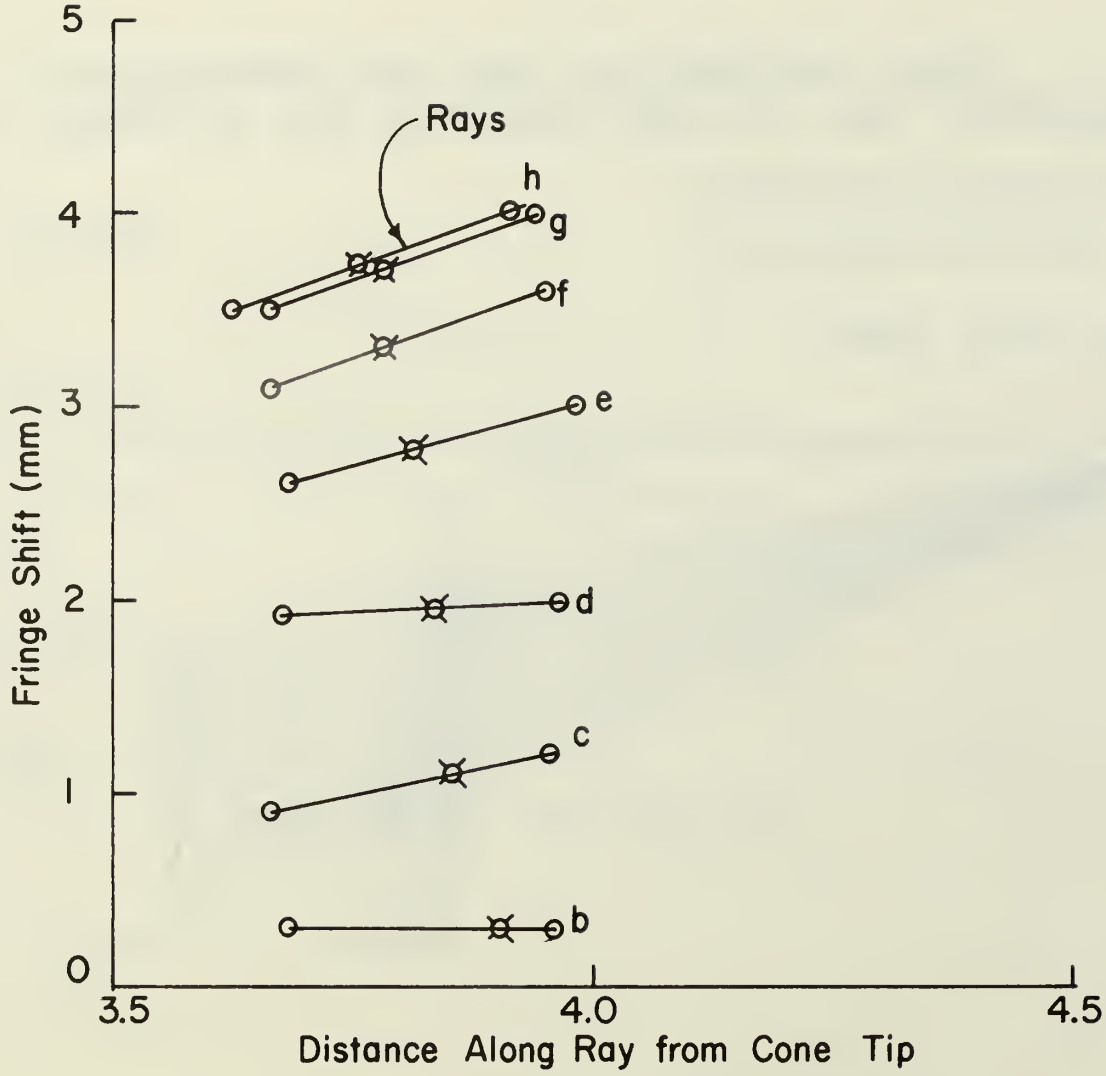


FIGURE 37. PLOT OF A TYPICAL FRINGE SHIFT VARIATION ALONG A RAY TO OBTAIN THE FRINGE SHIFT AT THE SECTION

TABLE I

VARIATION OF FRINGE SHIFT WITH DISTANCE
ALONG RAYS FROM THE CONE VERTEX

x = Fringe shift (mm.)

y = Distance from cone vertex along a ray (ins.)

Ray	Fringe 'a'		Fringe 'b'		Fringe 'c'		Fringe 'd'		Fringe 'e'		Section	
	x	y	x	y	x	y	x	y	x	y	x	y
1	0.0	4.50	0.0	4.63	0.0	4.75	0.0	4.89	0.0	5.0		4.48
2	2.0	4.37	2.1	4.49	1.9	4.63	1.7	4.77	1.0	4.92		4.44
3	3.3	4.26	5.5	4.40	3.6	4.50	3.2	4.64	2.1	4.78		4.40
4	4.1	4.17	4.4	4.29	4.4	4.41	4.1	4.53	3.5	4.68		4.36
5	4.9	4.08	4.9	4.21	5.4	4.32	5.0	4.43	4.7	4.58		4.32
6	5.5	4.02	5.6	4.13	6.1	4.24	5.5	4.35	5.6	4.48		4.28
7	6.0	3.95	6.1	4.08	6.6	4.18	6.0	4.29	6.4	414		4.26
8	6.4	3.91	6.8	4.03	7.2	4.13	6.7	4.22	6.9	4.34		4.23
9	6.6	3.87	7.2	3.97	7.7	4.08	6.6	4.18	7.4	4.28		4.21
10	6.6	3.84	7.3	3.92	7.8	4.03	6.6	4.14	7.7	4.23		4.18
11	6.5	3.80	7.1	3.91	7.2	4.0	6.5	4.10	7.7	4.19		4.15
12	6.3	3.79	6.4	3.90	7.1	3.99	6.3	4.08	7.3	4.17		4.14

TABLE II

RADIAL DISTANCE FROM THE CONE AXIS OF THE INTERSECTION OF RAYS WITH THE SECTION

Ray	Distance from cone axis	Normalized distance from cone axis
1	1.83	0.95
2	1.72	0.89
3	1.63	0.846
4	1.53	0.794
5	1.43	0.742
6	1.33	0.69
7	1.23	0.639
8	1.12	0.581
9	1.02	0.53
10	0.92	0.478
11	0.82	0.426
12	0.72	0.374

TABLE III

CALCULATION OF THE RADIAL VARIATION
OF FRINGE NUMBER AT THE SECTION

Average fringe spacing in the freestream = 2.775 mm.

Ray	Normalized distance from cone axis	Fringe shift (mm.)	Fringe number
1	0.95	0.0	0.0
2	0.89	2.08	0.75
3	0.846	3.5	1.26
4	0.794	4.42	1.59
5	0.742	5.4	1.946
6	0.69	5.92	2.13
7	0.639	6.16	2.22
8	0.581	6.69	2.41
9	0.53	6.73	2.425
10	0.478	6.87	2.476
11	0.426	7.22	2.60
12	0.374	6.80	2.45

APPENDIX A

REDUCTION OF AN INTERFEROGRAM TO OBTAIN FRINGE SHIFT DATA

The reduction of only one side of the interferogram obtained for the axisymmetric case is indicated as an illustration of the procedure employed. After projection of the negative, the cone surface, the shock waves, the grid line at the section concerned and the fringes from the projected image were traced on a sheet of paper as shown in Figure 36. The distance between the cone surface and the shock at the section was divided into a convenient number of parts and rays drawn from the tip of the cone through these points. The fringe shift distances at the point of intersection of each displaced fringe and the rays were measured by means of a 7X PEAK scale magnifier as well as the average fringe spacing in the freestream. The radial distance from the cone axis to the intersection of each of the rays with the section was also measured as also the distance along each ray to the intersection with each fringe and with the section concerned. A tabulation of these results is shown in Tables 1, 2, and 3. For each ray, the fringe shift distance was then plotted against the distance along the ray as in Figure 37, and from these curves the fringe shift distance at the section was obtained. These fringe shift distances were divided by the average fringe spacing in the freestream to obtain the fringe numbers at the various

points concerned. From a measurement of the distance from the tip of the cone to the section on the projected image and comparison with the known distance, the magnification of the image with respect to the actual conditions existing in the wind tunnel was determined. The radius of the inversion circle was obtained by assuming the distance of the shock from the cone axis to be at 95% of the radius of the inversion circle. The distance from the cone axis of the various intersections of the rays from the cone vertex and the section were obtained as a fraction of the inversion circle radius. The fringe number at these points was then plotted against the non-dimensional distance from the cone axis as shown in Figure 17 and a smooth curve drawn through these points. The value of the fringe number in the region occupied by the cone was taken as a constant value equal to that at the cone surface. From the curve so obtained, values of the fringe number at 101 equidistant points were obtained on each side of the cone axis (including the value at the axis) so that a total of 201 data values resulted. This fringe data was read into Subroutine READ of computer program HOLOFER and inverted to obtain the required density field.

The use of 201 data points was essentially dictated by the necessity to be able to define the shock position accurately and to have the fringe distribution in the region between the shock and the cone described fairly well. Since

the fringe curve from which these points were obtained was plotted using only about 10 experimental fringe values, the use of 201 points does not imply a higher accuracy in the density distribution output from the computer program.

APPENDIX B

USE OF COMPUTER PROGRAM "HOLOFER"

This computer program is designed to invert the array of fringe numbers to obtain the associated density field using the inversion first proposed by C. D. Maldonado and described in Section III of this report. The computer program can be run in basically 3 modes as described below:

(a) Mode 1

This mode provides the basic testing capability of the program and uses various test functions listed in Subroutine FUNCT in order to generate a G array which is then inverted back to obtain the original input function. Functions other than those specified in FUNCT can also be read in on cards by specifying a test function number of 8.0 in the original list of 42 parameters read in into the main program HOLOVERT. In this case the main program first calls Subroutine FREAD to read in the data cards. The first card in this data deck consists of two values indicating the total number of cards to be read in and the 'Z' section at which inversion is being performed and is input according to format 89 of the subroutine. Thence follows one point per card according to format 88 representing the numerical test function being input. This mode was employed in the present case for inverting the 201 points that were obtained from the AGARD tables by interpolation using Computer Program 1.

Apart from an evaluation of the effect of discontinuities due to the shock and the cone, this method provided advance information on the magnitude of the fringe numbers to be expected in the experimental results.

(b) Mode 2

This mode obtains the G array at regular intervals from irregularly spaced values of the fringe number function by utilizing Subroutine SHEET. The fringe data may also be simulated by specifying NCODE=1 in which case one of the functions in FUNCT may be used to generate the G array. This mode has not been applied to the present investigation.

(c) Mode 3

In this mode, the fringe numbers are read in directly at regularly spaced points by Subroutine READ which is called by Subroutine GARRAY. The various parameters in the first two cards of Subroutine READ serve to identify the symmetry of the fringe field. The following parameters have been used in the cases dealt with in this report:

Parameter	Axisymmetric Case	Asymmetric Case
NOF	Any value specifying the run number.	Any value specifying the run number
IMAX	201	201
JMAX	1	24
ISYM	2	1
JSYM	101	1
IMS	101	201
JMS	1	1
Z	The 'Z' section at which inversion is performed.	The 'Z' section at which inversion is performed.

Parameter	Axisymmetric Case	Asymmetric Case
XO	0.	0.
YO	0.	0.
PHISYM	0.	0.

Further details regarding the computer program are contained in Reference 3. A listing of the program is, however, included in this appendix for reference.

```
//  EXEC FORTCLGP,TIME.FORT=2,TIME.GO=120,REGION.GO=270K
//FORT.SYSIN DD *
C                                                                       CAL00010
C    ***************************HOLOVERT*******************************  CAL00020
C                                                                       CAL00030
C    HOLOVERT INVERTS THE  FIELD AT AN ARRAY OF POINTS IN A VARIABLE    CAL00040
C    COORDINATE SYSTEM.  IT SURVEYS NPTS POINTS EACH FOR A SET OF NLINS CAL00050
C    LINES ACROSS THE FIELD.                                            CAL00060
C                                                                       CAL00070
      COMMON IMAX,JMAX,IIMX,JJMX,IJMX,ALPHA,SIZE,EPS,MODE,BOX,SD,IX,Z   CAL00080
      COMMON /TAB/ INDEX,KEXTRA,MEXTRA,KLIMIT,MLIMIT,KOUT,MOUT         CAL00090
      COMMON /TAB2/ IPT,KPT,LPT,BND,NPTS,NLINS,RHOINF,RLAMDA,BETA       CAL00100
      COMMON /OUT/ XP,THEO,CALC,ERR,RHO,CA,FA                           CAL00110
      COMMON /EQPARA/ A,B,C,D,E,P,Q,S,T,U,V,W,RO,RA,NO,NA,NOF,NAF       CAL00120
      COMMON /SYM/ ISYM,JSYM,MSYM,FCU,IMS,JMS,QSYM                      CAL00130
      COMMON /IO/ CMS,IN1,IN2,IN4                                       CAL00140
      DATA BL,PL,ST,EX,OH,SC,DH,BR/1H ,1H+,1H*,1HX,1HO,1H:,1H-,1H|/     CAL00150
      DIMENSION RB(7),TL(62),RO(101),RA(101)                            CAL00160
C-----------------------------------------------------------------------CAL00170
      DIMENSION G(5151),GA(5151),H(202,5),SCF(73,6),BDA(4000)           CAL00180
      DIMENSION THEO(51,11),CALC(51,11),ERR(51),RHO(51)                 CAL00190
      DIMENSION CA(51),FA(51,11),AR(42),XP(51),YP(51)                   CAL00200
      CMS=0.                                                            
C-----------------------------------------------------------------------CAL00220
      REWIND 3                                                          
      IN1=5                                                             CAL00230
      IN2=5                                                             CAL00240
      IN4=5                                                             CAL00250
      IF (CMS.NE.1.) GO TO 20                                           CAL00260
      IN1=1                                                             CAL00270
      IN2=2                                                             CAL00280
      IN4=4                                                             CAL00290
      IF (CMS.EQ.1.) REWIND 4                                           CAL00300
20    READ (IN4,89) (AR(I),I=1,42)                                      CAL00310
      IMAX=AR(1)                                                        CAL00320
      JMAX=AR(2)*2.                                                     CAL00330
      IF (JMAX.LE.0) JMAX=1                                             CAL00340
      KLIMIT=AR(3)                                                      CAL00350
      MLIMIT=AR(4)                                                      CAL00360
      KEXTRA=AR(5)                                                      CAL00370
      MEXTRA=AR(6)                                                      CAL00380
      JSYM=AR(14)                                                       CAL00390
      ALPHA=AR(7)                                                       CAL00400
      SIZE=AR(8)                                                        CAL00410
      EPS=AR(9)                                                         CAL00420
      MODE=AR(13)                                                       CAL00430
      DGN=AR(17)                                                        CAL00440
      RHOINF=AR(10)                                                     CAL00450
```

```
      RLAMDA=AR(11)*1.E-8                                                    CAL00460
      BETA=AR(12)                                                            CAL00470
      NPTS=AR(15)                                                            CAL00480
      NLINS=AR(16)                                                           CAL00490
      SD=AR(18)                                                              CAL00500
      PHIZ=AR(19)                                                            CAL00510
      DELPHI=AR(20)                                                          CAL00520
      YPZERO=AR(21)                                                          CAL00530
      YPRNG=AR(22)                                                           CAL00540
      XPZERO=AR(23)                                                          CAL00550
      XPRNG=AR(24)                                                           CAL00560
      NOF=AR(25)                                                             CAL00570
      NAF=AR(26)                                                             CAL00580
      IPT=AR(27)                                                             CAL00590
      KPT=AR(28)                                                             CAL00600
      LPT=AR(29)                                                             CAL00610
      BND=AR(30)                                                             CAL00620
      A=AR(31)                                                               CAL00630
      B=AR(32)                                                               CAL00640
      C=AR(33)                                                               CAL00650
      D=AR(34)                                                               CAL00660
      E=AR(35)                                                               CAL00670
      P=AR(36)                                                               CAL00680
      S=AR(37)                                                               CAL00690
      T=AR(38)                                                               CAL00700
      U=AR(39)                                                               CAL00710
      V=AR(40)                                                               CAL00720
      W=AR(41)                                                               CAL00730
      Q=AR(42)                                                               CAL00740
      WRITE(6,90)
      WRITE(6,98)  (ST,I=1,75)
      WRITE(6,91) (AR(I),I=1,6)
      WRITE(6,92) (AR(I),I=7,12)
      WRITE(6,93) (AR(I),I=13,18)
      WRITE(6,94) (AR(I),I=19,24)
      WRITE(6,95) (AR(I),I=25,30)
      WRITE(6,96) (AR(I),I=31,36)
      WRITE(6,97) (AR(I),I=37,42)
      WRITE(6,98)  (ST,I=1,75)
   90 FORMAT(/////,' THE INPUT PARAMETERS FOR THIS RUN ARE :',///)
   91 FORMAT (/5X,'*  IMAX   * JMAX/2  * KLIMIT  * MLIMIT  * KEXTRA  *',SET00310
     1' MEXTRA  *'/3X,6F10.0)
   92 FORMAT (/5X,'*  ALPHA  *  SIZE   *  EPS.   * RHO-INF * LAMBDA  *',SET00340
     1'  BETA   *'/3X,2F10.3,F11.6,2X,F8.7,F9.1,F11.6)                    SET
   93 FORMAT (/5X,'*  MODE   *  SYM.   * POINTS  *  LINES  * DIAGNOS *',SET00360
     1' STD.DEV *'/2X,5F10.0,F10.3)
   94 FORMAT (/5X,'* PHIZERO * DELPHI  * YPZERO  * YPRANGE * XPZERO  *',
     1' XPRANGE *'/4X,6F10.3)
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 95  FORMAT (/5X,'* TST FUN * ADD FUN * GARRAY  *  GRAPH  * LIN PRT *',SET00400
    1' MAP BND *'/2X,6F10.0)
 96  FORMAT (/5X,'*    A    *    B    *    C    *    D    *    E    *',SET00420
    1'    P    *'/4X,6F10.3)
 97  FORMAT (/5X,'*    S    *    T    *    U    *    V    *    W    *',SET00440
    1'    Q    *'/4X,6F10.3)
 98  FORMAT (/3X,75A1)
     IF (DGN.GE.4) WRITE (6,89) (AR(I),I=1,42)                          CAL00750
     NNN=2                                                              CAL00760
     IF (MODE.LT.0) NNN=1                                               CAL00770
     IF (MODE.GT.5) NNN=3                                               CAL00780
     IF (MODE.GT.5) MODE=MODE-10                                        CAL00790
     NGP=0                                                              CAL00800
     IF (KLIMIT.LT.KEXTRA) KEXTRA=KLIMIT                                CAL00810
     IF (MLIMIT.LT.MEXTRA) MEXTRA=MLIMIT                                CAL00820
     IF (IPT.LT.0) NGP=IPT                                              CAL00830
     IF (IPT.LT.0) IPT=-IPT                                             CAL00840
     ISYM=2.1-(FLOAT(JSYM)/2.-FLOAT(JSYM/2))*2                          CAL00850
     IF (JSYM.EQ.0) ISYM=1                                              CAL00860
     IF (JSYM.GT.JMAX) ISYM=2                                           CAL00870
     IF (ISYM.EQ.1) JMAX=((JMAX+1)/2)*2                                 CAL00880
     RJMX=JMAX                                                          CAL00890
     MSYM=JSYM                                                          CAL00900
     IF ((MSYM.EQ.0).OR.(MSYM.GT.JMAX)) MSYM=1                          CAL00910
     FCU=ISYM*JSYM*JMAX                                                 CAL00920
     IF ((JSYM.GT.JMAX).OR.(JSYM.EQ.0)) FCU=JMAX                        CAL00930
     QSYM=FCU/RJMX                                                      CAL00940
     IMS=(IMAX+ISYM-1)/ISYM                                             CAL00950
     JMS=JMAX                                                           CAL00960
     IF (ISYM.EQ.1) JMS=(JMAX/2+1)/2                                    CAL00970
     IF (JSYM.EQ.0) JMS=JMAX/2                                          CAL00980
     MODE=ABS(AR(13))                                                   CAL00990
     XO=0.                                                              CAL01000
     YO=0.                                                              CAL01010
     ZD=0.                                                              CAL01020
     PHISYM=0.                                                          CAL01030
     HS=SIZE/2.                                                         CAL01040
     RHOS=1.286                                                         CAL01050
     BOX=RHOINF*BETA/RHOS/RLAMDA                                        CAL01060
     RPTS=NPTS                                                          CAL01070
     XPR=0.                                                             CAL01080
     IF (NPTS.GT.1) XPR=XPRNG/(RPTS-1.)/2.                              CAL01090
     XPM=-XPR                                                           CAL01100
     PIE=3.141592653589793                                              CAL01110
     MONE=1                                                             CAL01120
       WRITE (6,58) IMAX,JMAX,IMS,JMS,ISYM,JSYM,MSYM,QSYM,FCU           CAL01130
58     FORMAT (3X,'IMAX,JMAX,IMS,JMS,ISYM,JSYM,MSYM,QSYM,FCU',/         CAL01140
    1   7I5,2F7.3/)                                                     CAL01150
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      NTWO=2                                                                CAL01160
      IX=IMAX+JMAX                                                          CAL01170
      NF=IN1                                                                CAL01180
      IF ((MODE.EQ.1.).AND.(NOF.EQ.8).AND.(DGN.GE.1.)) WRITE (6,69)         CAL01190
      IF ((MODE.EQ.1).AND.(NOF.EQ.8)) CALL FREAD (NO,RO,NF,ZD)              CAL01200
      Z=ZD                                                                  CAL01210
      IF (DGN.GE.1.) WRITE (6,68)                                           CAL01220
      CALL GARRAY (G,GA,NOF,DGN,MONE,XO,YO,PHISYM)                          CAL01230
      LM=1                                                                  CAL01240
      IF ((LPT.EQ.0).AND.(BND.EQ.0)) LM=0                                   CAL01250
      IIMX=IMAX+1                                                           CAL01260
      JJMX=JMAX+1                                                           CAL01270
      IJMX=IMAX*JMAX                                                        CAL01280
      NBD=1                                                                 CAL01290
      IF (JSYM.EQ.0) NBD=2                                                  CAL01300
      KBD=KLIMIT*NBD                                                        CAL01310
      DO 15 IJ=1,IJMX                                                       CAL01320
15    GA(IJ)=0.                                                             CAL01330
      IF (NAF.EQ.0) GO TO 16                                                CAL01340
      NF=IN2                                                                CAL01350
      IF ((NAF.EQ.8).AND.(DGN.GE.1.)) WRITE (6,69)                          CAL01360
      IF (NAF.EQ.8) CALL FREAD (NA,RA,NF,ZD)                                CAL01370
      MST=MODE                                                              CAL01380
      MODE=1                                                                CAL01390
      IF (DGN.GE.1.) WRITE (6,68)                                           CAL01400
      IF (NAF.NE.0) CALL GARRAY (GA,G,NAF,DGN,NTWO,XO,YO,PHISYM)            CAL01410
      MODE=MST                                                              CAL01420
      DO 6 IJ=1,IJMX                                                        CAL01430
6     G(IJ)=G(IJ)+GA(IJ)                                                    CAL01440
16    RLINS=NLINS                                                           CAL01450
      IF (NAF.EQ.8) WRITE (6,88) NA,(RA(L),L=1,NA)                          CAL01460
      IF (NOF.EQ.8) WRITE (6,87) NO,(RO(I),I=1,NO)                          CAL01470
      IF (LM.EQ.0) GO TO 14                                                 CAL01480
      RB(1)=-1.                                                             CAL01490
      DO 1 I=2,7                                                            CAL01500
1     RB(I)=RB(I-1)+.5                                                      CAL01510
      TPIE=2.*PIE                                                           CAL01520
      MPIE=-PIE                                                             CAL01530
      DYP=0.                                                                CAL01540
      DXP=0.                                                                CAL01550
      IF (NLINS.GT.1) DYP=YPRNG/(RLINS-1.)                                  CAL01560
      IF (NPTS.GT.1) DXP=XPRNG/(RPTS-1.)                                    CAL01570
      IF ((DGN.GE.1.).AND.(NNN.EQ.2)) WRITE (6,64)                          CAL01580
      IF(NNN.EQ.2) CALL BDGEN (G,H,SCF,DGN,NBD,BDA,KBD)                     CAL01590
      DO 5 J=1,NLINS                                                        CAL01600
      IF (DGN.GE.1.) WRITE (6,67) J                                         CAL01610
      RJM=J-1                                                               CAL01620
      PHI=PHIZ+DELPHI*RJM                                                   CAL01630
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      YP(J)=YPZERO+DYP*RJM                                                  CAL01640
      PSI=(PHI+90.)*PIE/180.                                                CAL01650
      TAU=PSI-PHISYM                                                        CAL01660
      IF (LPT.EQ.0) GO TO 9                                                 CAL01670
      IF (LPT.LE.1) WRITE (6,78) (ST,I=1,124)                               CAL01680
      IF (LPT.GT.1) WRITE (6,74) (ST,I=1,95)                                CAL01690
      IF ((CMS.EQ.1.).AND.(LPT.GT.1)) READ (5,79) ZZ                        CAL01700
      WRITE (6,86)                                                          CAL01710
      WRITE (6,85) Z,PHI,YP(J)                                              CAL01720
      WRITE (6,76)                                                          CAL01730
      IF (MODE.EQ.1) WRITE (6,83) (RB(I),I=1,7)                             CAL01740
      IF (MODE.GT.1) WRITE (6,80) (RB(I),I=1,7)                             CAL01750
      WRITE (6,81) (DH,I=1,54),(PL,II=1,13)                                 CAL01760
9     IC=0                                                                  CAL01770
      DO 3 I=1,NPTS                                                         CAL01780
      RIM=I-1                                                               CAL01790
      THEO(I,J)=0.                                                          CAL01800
      CA(I)=0.                                                              CAL01810
      FA(I,J)=0.                                                            CAL01820
      ERR(I)=0.                                                             CAL01830
      CALC(I,J)=0.                                                          CAL01840
      RHO(I)=0.                                                             CAL01850
      XP(I)=XPZERO+DXP*RIM                                                  CAL01860
      XPII=ABS(XP(I))                                                       CAL01870
      IF (XPII.LT.1.E-10) XP(I)=0.                                          CAL01880
      RS=SQRT(XP(I)**2+YP(J)**2)/HS                                         CAL01890
      IF (RS.GT.1.) GO TO 13                                                CAL01900
      THT=ATANM(YP(J),XP(I))                                                CAL01910
      IF (XPII.EQ.0.) THT=0.                                                CAL01920
      SIG=TAU-PIE/2.+THT                                                    CAL01930
      IF (SIG.GT.PIE) SIG=SIG-TPIE                                          CAL01940
      IF (SIG.LT.MPIE) SIG=SIG+TPIE                                         CAL01950
      SIGI=SIG                                                              CAL01960
      XS=RS*COS(SIG)                                                        CAL01970
      IF (DGN.GE.1.) WRITE (6,44) SIG                                       CAL01980
44    FORMAT ('    SIG=',E10.3)                                             CAL01990
      YS=RS*SIN(SIG)                                                        CAL02000
      IF(DGN.GE.5) WRITE (6,57) PHI,DELPHI,PSI,TAU,THT,SIG,SIGI,XS,YS       CAL02010
57      FORMAT ('  ANGLES=',10E10.3)                                        CAL02020
      RI=I                                                                  CAL02030
      F=0.                                                                  CAL02040
      IF (DGN.GE.2.) WRITE (6,66) I                                         CAL02050
      CALL FUNCT (XS,YS,FA(I,J),NAF,DGN,NTWO)                               CAL02060
      IF (MODE.EQ.1) CALL FUNCT (XS,YS,F,NOF,DGN,MONE)                      CAL02070
      THEO(I,J)=F                                                           CAL02080
      IF (NNN.GE.2) REWIND 3                                                CAL02090
      IF(NNN.GE.2) CALL FIELD (RS,SIGI,SOLN,NBD,BDA,DGN,KBD)                CAL02100
      IF(NNN.EQ.1) CALL FIELD2 (RS,SIGI,SOLN,G,H,SCF,DGN)                   CAL02110
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      CA(I)=SOLN/BOX/HS                                                  CAL02120
      CALC(I,J)=CA(I)-FA(I,J)                                            CAL02130
      RHO(I)=RHOINF*(CALC(I,J)+1.)                                       CAL02140
      ERR(I)=CA(I)                                                       CAL02150
      IF (MODE.EQ.1) ERR(I)=(CALC(I,J)-THEO(I,J))                        CAL02160
      IF (MODE.GT.1) THEO(I,J)=FA(I,J)                                   CAL02170
      IF (LPT.EQ.0) GO TO 3                                              CAL02180
   13 LC=0                                                               CAL02190
      TL(1)=BL                                                           CAL02200
      TTL=0.                                                             CAL02210
      IF ((XP(I).GT.XPM).AND.(XP(I).LT.XPR)) TTL=1.
      IF (IC.EQ.5) IC=0
      IF (IC.EQ.0) TL(1)=PL
      DO 2 L=2,62
      TL(L)=BL                                                           CAL02260
      IF ((I.EQ.1).OR.(TTL.EQ.1.).OR.(I.EQ.NPTS)) TL(L)=PL               CAL02270
      IF (LC.EQ.10) LC=0                                                 CAL02280
      IF ((IC.EQ.0).AND.(LC.EQ.0)) TL(L)=PL                              CAL02290
    2 LC=LC+1                                                            CAL02300
      TL(2)=PL                                                           CAL02310
      TL(22)=PL                                                          CAL02320
      TL(62)=PL                                                          CAL02330
      IC=IC+1                                                            CAL02340
      RLW=(CA(I)+1.)*20.+2.5                                             CAL02350
      LW=RLW                                                             CAL02360
      IF (LW.GT.62) LW=62                                                CAL02370
      IF (LW.LT.2) LW=2                                                  CAL02380
      TL(LW)=SC                                                          CAL02390
      RLY=(FA(I,J)+1.)*20.+2.5                                           CAL02400
      LY=RLY                                                             CAL02410
      IF (LY.GT.62) LY=62                                                CAL02420
      IF (LY.LT.2) LY=2                                                  CAL02430
      IF (NAF.NE.0) TL(LY)=ST                                            CAL02440
      RLX=(THEO(I,J)+1.)*20.+2.5                                         CAL02450
      LX=RLX                                                             CAL02460
      IF (LX.GT.62) LX=62                                                CAL02470
      IF (LX.LT.2) LX=2                                                  CAL02480
      IF (MODE.EQ.1) TL(LX)=OH                                           CAL02490
      RLZ=(CALC(I,J)+1.)*20.+2.5                                         CAL02500
      LZ=RLZ                                                             CAL02510
      IF (LZ.GT.62) LZ=62                                                CAL02520
      IF (LZ.LT.2) LZ=2                                                  CAL02530
      TL(LZ)=EX                                                          CAL02540
      WRITE (6,82)MOUT,KOUT,INDEX,THEO(I,J),ERR(I),CALC(I,J),RHO(I),     CAL02550
     1XP(I),(TL(L),L=1,62)                                               CAL02560
      IF ((NPTS.LE.20).AND.(I.NE.NPTS)) WRITE (6,79)                     CAL02570
    3 CONTINUE                                                           CAL02580
      IF (LPT.NE.0) WRITE (6,81) (DH,I=1,54),(PL,I=1,13)                 CAL02590
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      TMAX=0.                                                          CAL02600
      TMIN=0.                                                          CAL02610
      IE=0                                                             CAL02620
      BE=0.                                                            CAL02630
      EB=0.                                                            CAL02640
      DO 4 I=1,NPTS                                                    CAL02650
      TH=THEO(I,J)                                                     CAL02660
      IF (TH.GT.TMAX) TMAX=TH                                          CAL02670
      IF (TH.LT.TMIN) TMIN=TH                                          CAL02680
      ER=ABS(CALC(I,J)-TH)                                             CAL02690
      IF (ER.LE.BE) GO TO 4                                            CAL02700
      BE=ER                                                            CAL02710
      IE=I                                                             CAL02720
4     CONTINUE                                                         CAL02730
      TMM=TMAX-TMIN                                                    CAL02740
      EB=RHOINF*(CALC(IE,J)-THEO(IE,J))                                CAL02750
      IF (TMM.NE.0.) BE=(CALC(IE,J)-THEO(IE,J))*100./TMM               CAL02760
      IF ((MODE.EQ.1).AND.(LPT.NE.0)) WRITE (6,75) EB,XP(IE),BE        CAL02770
      IF (DELPHI.NE.0.) YP(J)=PHI                                      CAL02780
5     CONTINUE                                                         CAL02790
      IF (BND.EQ.0.) GO TO 14                                          CAL02800
      IF (LPT.EQ.1) WRITE (6,78) (ST,I=1,124)                          CAL02810
      IF (LPT.GT.1) WRITE (6,74) (ST,I=1,95)                           CAL02820
      IF ((CMS.EQ.1.).AND.(LPT.GT.1)) READ (5,79) ZZ                   CAL02830
      IF (DGN.GE.1.) WRITE (6,63)                                      CAL02840
      CALL MAP (NPTS,NLINS,CALC,NOF,Z,BND)                             CAL02850
14    IF (NAF.EQ.0) GO TO 10                                           CAL02860
      NAO=10*NOF+NAF                                                   CAL02870
      IF ((DGN.GE.1.).AND.(NGP.EQ.-3)) WRITE (6,62)                    CAL02880
      IF (NGP.EQ.-3) CALL GPUNCH (Z,XO,YO,PHISYM,NAO,IMAX,JMAX,G)      CAL02890
      DO 7 IJ=1,IJMX                                                   CAL02900
7     G(IJ)=G(IJ)-GA(IJ)                                               CAL02910
10    IF (IPT.LE.0) GO TO 11                                           CAL02920
      IF ((IPT.EQ.1).OR.(IPT.EQ.3)) WRITE (6,78) (ST,I=1,124)          CAL02930
      IF ((IPT.EQ.2).OR.(IPT.GE.4)) WRITE (6,74) (ST,I=1,95)           CAL02940
      IF ((CMS.EQ.1.).AND.((IPT.EQ.2).OR.(IPT.GE.4))) READ (5,79) ZZ   CAL02950
      CALL GPRINT (G,MONE)                                             CAL02960
      IF (NGP.EQ.-1) CALL GPUNCH (Z,XO,YO,PHISYM,NOF,IMAX,JMAX,G)      CAL02970
      IF (IPT.EQ.3) WRITE (6,78) (ST,I=1,124)                          CAL02980
      IF (IPT.GE.4) WRITE (6,74) (ST,I=1,95)                           CAL02990
      IF ((CMS.EQ.1.).AND.(IPT.GE.4)) READ (5,79) ZZ                   CAL03000
      IF (IPT.GE.3) CALL GPRINT (GA,NTWO)                              CAL03010
11    IF (KPT.LE.0) GO TO 12                                           CAL03020
      IF ((KPT.EQ.1).OR.(KPT.EQ.3)) WRITE (6,78) (ST,I=1,124)          CAL03030
      IF ((KPT.EQ.2).OR.(KPT.GE.4)) WRITE (6,74) (ST,I=1,95)           CAL03040
      IF ((CMS.EQ.1.).AND.((KPT.EQ.2).OR.(KPT.GE.4))) READ (5,79) ZZ   CAL03050
      IF (DGN.GE.1.) WRITE (6,61)                                      CAL03060
      CALL GPLOT (G,GA,JMS)                                            CAL03070
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12    WRITE (6,78) (EX,I=1,124)                                          CAL03080
      AGAIN=ST                                                           CAL03090
      IF (CMS.NE.1.) READ(5,60) AGAIN                                    CAL03100
      IF (AGAIN.EQ.BL) GO TO 20                                          CAL03110
      WRITE (6,77)                                                       CAL03120
89    FORMAT (6F12.7)                                                    CAL03130
88    FORMAT (/'     THE INPUT DATA FOR ADD-ON FUNCTION NO.8 (',I3,      CAL03140
     1' POINTS) WAS:'/7(11F10.3/))                                       CAL03150
87    FORMAT (/'     THE INPUT DATA FOR THE TEST FUNCTION NO.8, (',I3,   CAL03160
     1' POINTS) WAS:'/7(11F10.3/))                                       CAL03170
86    FORMAT (1H1//'   THE INVERTED CROSS SECTION FOR:')                 CAL03180
85    FORMAT (10X,'Z  =',F8.3,' CM.'/10X,'PHI=',F8.3,' DEGREES'/10X,     CAL03190
     14HY' =,F8.3,' CM.',44X,'0 = ORIGINAL FUNCTION'/                    CAL03200
     270X,'* = ADD-ON FUNCTION')                                         CAL03210
84    FORMAT (' ADJUST PAGE, HIT SPACE AND RETURN.')                     CAL03220
83    FORMAT ('   LIMIT  MAX  ORIGINAL     ABS.  COMPUTED (MG/CC)'/'    MCAL03230
     1   K TERM  FUNCTION    ERROR  FUNCTION DENSITY ',6H  X'  ,F4.1,    CAL03240
     26F10.1)                                                            CAL03250
82    FORMAT (2X,I2,1X,I3,1X,I4,1X,F9.4,2X,F7.3,1X,F9.4,1X,F7.3,         CAL03260
     1F7.3,1X,62A1)                                                      CAL03270
81    FORMAT (3X,54A1,2X,A1,12(4X,A1))                                   CAL03280
80    FORMAT ('   LIMIT  MAX',3X,'ADD-ON',6X,'THE',4X,'DENSITY (MG/CC)'/CAL03290
     1'    M   K TERM',2X,'FUNCTION',5X,'SUM',3X,'FUNCTION DENSITY ',    CAL03300
     26H  X'  ,F4.1,6F10.1)                                              CAL03310
79    FORMAT (1X,F10.3)                                                  CAL03320
78    FORMAT (1X,124A1)                                                  CAL03330
77    FORMAT (1X//)                                                      CAL03340
76    FORMAT (4X,'FUNCTION = (RHO/RHO-INFINITY)-1.0',33X,': = THE'       CAL03350
     1,' INVERTED SUM'/70X,'X = COMPUTED FUNCTION')                      CAL03360
75    FORMAT ('     LARGEST ERROR: ',F8.6,' GMS/CC;      AT ',3HX'=,F6.3CAL03370
     1,';',4X,F10.2,' PERCENT.'//)                                       CAL03380
74    FORMAT (1X,47A1,' SET PAGE, HIT SPACE, RETURN ',48A1)              CAL03390
69    FORMAT ('   CALL FREAD')                                           CAL03400
68    FORMAT ('   CALL GARRAY')                                          CAL03410
67    FORMAT ('   LINE',I3,' DO LOOP')                                   CAL03420
66    FORMAT ('   POINT',I3,' CALL FUNCT')                               CAL03430
65    FORMAT ('   CALL FIELD')                                           CAL03440
64    FORMAT ('   CALL BDGEN')                                           CAL03450
63    FORMAT ('   CALL MAP')                                             CAL03460
62    FORMAT ('   CALL GPUNCH')                                          CAL03470
61    FORMAT ('   CALL GPLOT')                                           CAL03480
60    FORMAT (80A1)                                                      CAL03490
      STOP                                                               CAL03500
      END                                                                CAL03510
                                                                         CAL03520
C000001                                                                  SUB00010
C                                                                        SUB00020
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      SUBROUTINE BDGEN (G,H,SCF,DGN,NBD,BDA,KBD)                        SUB00030
C                                                                       SUB00040
C   BDGEN EVALUATES THE B AND D COEFFICIENTS FOR ALL M AND K, AND WRITES SUB00050
C   THE ARRAY LINEARLY ON DISK.                                         SUB00060
C                                                                       SUB00070
      COMMON IMAX,JMAX,IIMX,JJMX,IJMX,ALPHA,SIZE,EPS,MODE,BOX,SD,IX,Z   SUB00080
      COMMON /TAB/ INDEX,KEXTRA,MEXTRA,KLIMIT,MLIMIT,KOUT,MOUT          SUB00090
      COMMON /SYM/ ISYM,JSYM,MSYM,FCU,IMS,JMS,QSYM                      SUB00100
      DIMENSION G(IJMX),H(IIMX,5),SCF(JJMX,6),BDA(KBD)                  SUB00110
C   INITIALIZE THE VALUES:                                              SUB00120
      INDEX=0                                                           SUB00130
      KL2=NBD*KLIMIT                                                    SUB00140
      REWIND 3                                                          SUB00150
      JJMX6=JJMX*6                                                      SUB00160
      IIMX2=(IIMX+1)/2                                                  SUB00170
      PIE=3.141592653589793                                             SUB00180
      RIMAX=IMAX                                                        SUB00190
      KLMP=KLIMIT+1                                                     SUB00200
      DX=2./RIMAX                                                       SUB00210
      RJMAX=JMAX                                                        SUB00220
      DXI=2.*PIE/FCU                                                    SUB00230
C   INITIALIZE THE MODIFIED HERMITE POLYNOMIAL ARRAY;  VECTORS:         SUB00240
C   (1)=H1, (2)=H2, (3)=ALPHA*X(I), (4)=HM+2 STORED, (5)=HM+1 STORED    SUB00250
      DO 1 II=1,IIMX2                                                   SUB00260
      RII=II                                                            SUB00270
      IIM=IIMX-II+1                                                     SUB00280
      H(II,3)=ALPHA*(RII*DX-DX-1.)                                      SUB00290
      H(IIM,3)=-H(II,3)                                                 SUB00300
      H(II,1)=2.*H(II,3)                                                SUB00310
      H(II,2)=(H(II,3)*H(II,1)-1.)/3.                                   SUB00320
      H(IIM,1)=-H(II,1)                                                 SUB00330
      H(IIM,2)=H(II,2)                                                  SUB00340
      H(II,5)=H(II,2)                                                   SUB00350
      H(IIM,5)=H(IIM,2)                                                 SUB00360
      H(II,4)=H(II,1)                                                   SUB00370
1     H(IIM,4)=H(IIM,1)                                                 SUB00380
      SIGN=1.                                                           SUB00390
C   INITIATE THE SIN/COS ARRAY:                                         SUB00400
      DO 2 J=1,JJMX                                                     SUB00410
      RJM=J-1                                                           SUB00420
      SCF(J,1)=0.                                                       SUB00430
      SCF(J,2)=1.                                                       SUB00440
      SCF(J,3)=SIN(RJM*DXI-PIE/2.)                                      SUB00450
      SCF(J,4)=COS(RJM*DXI-PIE/2.)                                      SUB00460
      SCF(J,5)=0.                                                       SUB00470
2     SCF(J,6)=0.                                                       SUB00480
      MS=0                                                              SUB00490
C   COMMENCE THE M LOOP:                                                SUB00500
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      DO 7 MP=1,MLIMIT                                                  SUB00510
      M=MP-1                                                            SUB00520
      RM=M                                                              SUB00530
      SIGN=-SIGN                                                        SUB00540
      IF (DGN.LE.-4) WRITE (6,88) SCF(1,1),SCF(2,1),SCF(1,2),SCF(2,2)   SUB00550
C   TEST FOR SYMMETRY SKIPS:                                            SUB00560
      IF (MS.EQ.MSYM) MS=0                                              SUB00570
      TOTAL=0.                                                          SUB00580
      MS=MS+1                                                           SUB00590
      IF (MS.NE.1) GO TO 6                                              SUB00600
C   COMMENCE THE K LOOP:                                                SUB00610
      DO 5 KP=1,KLIMIT                                                  SUB00620
      K=KP-1                                                            SUB00630
      PK=KP                                                             SUB00640
      RK=K                                                              SUB00650
      INDEX=INDEX+1                                                     SUB00660
C   CALL THE B & D COEFFICIENTS AND WRITE THEM ON DISK:                 SUB00670
      CALL BD (M,K,G,H,SCF,B,D,JJMX6)                                   SUB00680
      IF(DGN.EQ.3.) WRITE(6,89) M,K,B,D
      IF (DGN.LE.-2) WRITE (6,89) M,K,B,D                               SUB00690
      IF (DGN.LE.-4) WRITE (6,88) H(1,1),H(1,2),H(1,4),H(1,5)           SUB00700
      KK=K*NBD+1                                                        SUB00710
      K2=KP*NBD                                                         SUB00720
      BDA(K2)=D                                                         SUB00730
      BDA(KK)=B                                                         SUB00740
C   GENERATE THE NEXT ORDER OF THE SET OF HERMITE POLYNOMIALS FOR NEW K:SUB00750
      ORDER=M+2*KP+1                                                    SUB00760
      HA=SQRT(PK*(PK+RM))/ORDER                                         SUB00770
      HB=2.*SQRT((PK+1.)*(RM+PK+1.))/(ORDER+1.)/(ORDER+2.)              SUB00780
      DO 5 II=1,IIMX2                                                   SUB00790
      IIM=IIMX-II+1                                                     SUB00800
      H(II,1)=2.*(H(II,3)*H(II,2)-HA*H(II,1))                           SUB00810
      H(IIM,1)=SIGN*H(II,1)                                             SUB00820
5     H(II,2)=HB*(H(II,3)*H(II,1)-ORDER*H(II,2))                        SUB00830
C   ADVANCE THE SIN/COS ARRAY FOR THE NEXT M:                           SUB00840
      DO 3 J=1,JJMX                                                     SUB00850
      IF (DGN.LE.-5) WRITE (6,87) (SCF(J,NT),NT=1,6)                    SUB00860
87      FORMAT ('  SIN/COS MXI:'8E10.3)                                SUB00870
      STEMP=SCF(J,1)                                                    SUB00880
      SCF(J,1)=SCF(J,1)*SCF(J,4)+SCF(J,2)*SCF(J,3)                      SUB00890
3     SCF(J,2)=SCF(J,2)*SCF(J,4)-STEMP*SCF(J,3)                         SUB00900
      DO 4 J=1,JMAX                                                     SUB00910
      SCF(J,5)=SCF(J+1,1)-SCF(J,1)                                      SUB00920
4     SCF(J,6)=SCF(J+1,2)-SCF(J,2)                                      SUB00930
      WRITE (3) (BDA(I),I=1,KBD)                                        SUB00940
      IF (DGN.LE.-3) WRITE (6,88) (BDA(I),I=1,10)                       SUB00950
      IF (JSYM.GT.JMAX) RETURN                                          SUB00960
6     RM=RM+1                                                           SUB00970
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C    REGENERATE THE HERMITE ARRAY FOR NEW M, K=0:                                    SUB00980
       DO 7 II=1,IIMX2                                                               SUB00990
       IIM=IIMX-II+1                                                                 SUB01000
       H(II,2)=H(II,4)*SQRT(RM)/(RM+1.)                                              SUB01010
       H(II,1)=H(II,5)*(RM+2.)                                                       SUB01020
       H(IIM,1)=-SIGN*H(II,1)                                                        SUB01030
       H(II,2)=2.*SQRT(RM+1)*(H(II,3)*H(II,1)-(RM+1.)*H(II,2))                       SUB01040
       H(II,2)=H(II,2)/(RM+2.)/(RM+3.)                                               SUB01050
       H(II,4)=H(II,1)                                                               SUB01060
7      H(II,5)=H(II,2)                                                               SUB01070
89     FORMAT ('    M=',I4,',    K=',I4,',    B=',E10.4,',    D=',E10.4)           SUB01080
88     FORMAT (2X,10E10.3)                                                           SUB01090
       RETURN                                                                        SUB01100
       END                                                                           SUB01110
C000002                                                                              SUB01120
C                                                                                    SUB01130

       SUBROUTINE FIELD (RS,SIG,SOLN,NBD,BDA,DGN,KBD)                                SUB01140
C                                                                                    SUB01150
C    FIELD EVALUATES THE VALUE OF THE FIELD FUNCTION AT A PARTICULAR                 SUB01160
C    POINT DESIGNATED IN CYLINDRICAL COORDINATES, BY USING THE INVERSION             SUB01170
C    EQUATION OF MALDONADO, ET.AL.  FIELD USES THE ARRAY OF  B & D                   SUB01180
C    COEFFICIENTS GENERATED IN SUBROUTINE BDGEN.                                     SUB01190
C                                                                                    SUB01200
       COMMON IMAX,JMAX,IIMX,JJMX,IJMX,ALPHA,SIZE,EPS,MODE,BOX,SD,IX,Z               SUB01210
       COMMON /TAB/ INDEX,KEXTRA,MEXTRA,KLIMIT,MLIMIT,KOUT,MOUT                     SUB01220
       COMMON /SYM/ ISYM,JSYM,MSYM,FCU,IMS,JMS,QSYM                                  SUB01230
       DIMENSION BDA(KBD), STK(52), STM(52)                                          SUB01240
C    INITIALIZE THE VALUES:                                                          SUB01250
       INDEX=0                                                                       SUB01260
       MTIMER=0                                                                      SUB01270
       KOUT=0                                                                        SUB01280
       MOUT=0                                                                        SUB01290
       MMAX=0                                                                        SUB01300
       KMAX=0                                                                        SUB01310
       TOTAL=0.                                                                      SUB01320
       JJMX6=JJMX*6                                                                  SUB01330
       REWIND 3                                                                      SUB01340
       AR=ALPHA*RS                                                                   SUB01350
       ARG=AR**2                                                                     SUB01360
       EXPON=EXP(-ARG)                                                               SUB01370
       PIE=3.141592653589793                                                         SUB01380
       APP=ALPHA/PIE/PIE                                                             SUB01390
       M=0                                                                           SUB01400
       RM=M                                                                          SUB01410
       RIMAX=IMAX                                                                    SUB01420
       DX=2./RIMAX                                                                   SUB01430
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      RJMAX=JMAX                                                        SUB01440
      SIGN=1.                                                           SUB01450
      STK(1)=0.                                                         SUB01460
      STM(1)=0.                                                         SUB01470
      SMS=0.                                                            SUB01480
      CMS=1.                                                            SUB01490
      SMI=SIN(SIG)                                                      SUB01500
      CMI=COS(SIG)                                                      SUB01510
      MEP=MEXTRA+1                                                      SUB01520
      DO 16 MB=1,MEP                                                    SUB01530
16    STM(MB)=0.                                                        SUB01540
      FM=1.                                                             SUB01550
      MS=0                                                              SUB01560
C   COMMENCE THE M LOOP:                                                SUB01570
2     SIGN=-SIGN                                                        SUB01580
      K=0                                                               SUB01590
      RK=K                                                              SUB01600
      RM=M                                                              SUB01610
      ARM=1.                                                            SUB01620
      IF (M.NE.0) ARM=AR**M                                             SUB01630
      KTIMER=0                                                          SUB01640
      KEP=KEXTRA+1                                                      SUB01650
      DO 15 KB=1,KEP                                                    SUB01660
15    STK(KB)=0.                                                        SUB01670
      SIGNK=-1.                                                         SUB01680
C   COMPUTE THE K=0 & K=1 ORDERS OF LAGUERRE POLYNOMIAL FOR GIVEN M:    SUB01690
      PM=0.                                                             SUB01700
      P=SQRT(1./FM)                                                     SUB01710
      PP=(RM+1.-ARG)*SQRT(1./FM/(RM+1.))                                SUB01720
C   TEST FOR SYMMETRY SKIPS:                                            SUB01730
      IF (MS.EQ.MSYM) MS=0                                              SUB01740
      MS=MS+1                                                           SUB01750
      IF (MS.NE.1) GO TO 7                                              SUB01760
C   READ A LINE OF B & D COEFFICIENTS FOR GIVEN M:                      SUB01770
      READ (3) (BDA(I),I=1,KBD)                                         SUB01780
      IF (DGN.LE.-6) WRITE (6,88) (BDA(I),I=1,10)                       SUB01790
C   COMMENCE THE K LOOP:                                                SUB01800
3     INDEX=INDEX+1                                                     SUB01810
      SIGNK=-SIGNK                                                      SUB01820
C   COMPUTE THE M,K SUMMATION TERM:                                     SUB01830
      KK=K*NBD+1                                                        SUB01840
      B=BDA(KK)                                                         SUB01850
      D=0.                                                              SUB01860
      IF (NBD.EQ.2) D=BDA(KK+1)                                         SUB01870
      BRAKET=B                                                          SUB01880
      IF(RM.EQ.0.) GO TO 4                                              SUB01890
      BRAKET=B*CMS+D*SMS                                                SUB01900
4     ADD=SIGNK*BRAKET*P*ARM                                            SUB01910
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      TOTAL=TOTAL+ADD                                                    SUB01920
      IF (DGN.GT.-5) GO TO 5                                             SUB01930
      STOT=TOTAL*EXPON*APP/BOX/SIZE                                      SUB01940
      WRITE (6,89) M,K,STOT,ADD,BRAKET,P,ARM,B,CMS,D,SMS                 SUB01950
C     ESTABLISH CHECK AS THE RELATIVE SIZE OF THE M,K TERM OF THE SERIES: SUB01960
5     CHECK=ABS(ADD)                                                     SUB01970
      IF (TOTAL.GT.EPS) CHECK=ABS(ADD/TOTAL)                             SUB01980
C     ADVANCE THE K INDEX:                                               SUB01990
      K=K+1                                                              SUB02000
      RK=K                                                               SUB02010
      DO 10 KA=1,KEXTRA                                                  SUB02020
      KB=KEXTRA-KA+1                                                     SUB02030
10    STK(KB+1)=STK(KB)                                                  SUB02040
      STK(2)=TOTAL                                                       SUB02050
      ORDER=M+2*K+1                                                      SUB02060
C     GENERATE THE NEXT ORDER OF LAGUERRE POLYNOMIAL FOR NEW K:          SUB02070
      PM=P                                                               SUB02080
      P=PP                                                               SUB02090
      PP=P*(ORDER-ARG)-PM*SQRT(RK*(RM+RK))                               SUB02100
      PP=PP/SQRT((RK+1.)*(RM+RK+1.))                                     SUB02110
C     SET K TIMER TO PROVIDE EXTRA K TERMS AFTER CHECK < EPS:            SUB02120
      KTIMER=KTIMER+1                                                    SUB02130
      IF (K.GE.KLIMIT) GO TO 6                                           SUB02140
      IF (CHECK.GE.EPS) KTIMER=0                                         SUB02150
      IF (KTIMER.LE.KEXTRA) GO TO 3                                      SUB02160
      GO TO 7                                                            SUB02170
6     KOUT=KOUT+1                                                        SUB02180
      IF (KEXTRA.EQ.0) GO TO 7                                           SUB02190
      TOTAL=0.                                                           SUB02200
      DO 11 KA=1,KEXTRA                                                  SUB02210
11    TOTAL=TOTAL+STK(KA+1)                                              SUB02220
      RKX=KEXTRA                                                         SUB02230
      TOTAL=TOTAL/RKX                                                    SUB02240
C     END OF K LOOP;  ADVANCE M:                                         SUB02250
7     M=M+1                                                              SUB02260
      RM=M                                                               SUB02270
      STP=SMS                                                            SUB02280
      SMS=SMS*CMI+CMS*SMI                                                SUB02290
      CMS=CMS*CMI-STP*SMI                                                SUB02300
      IF (K.GT.KMAX) KMAX=K                                              SUB02310
      FM=FM*RM                                                           SUB02320
      DO 12 MA=1,MEXTRA                                                  SUB02330
      MB=MEXTRA-MA+1                                                     SUB02340
12    STM(MB+1)=STM(MB)                                                  SUB02350
      STM(2)=TOTAL                                                       SUB02360
C     SET M TIMER FOR EXTRA M TERMS:                                     SUB02370
      IF (MS.EQ.1) MTIMER=MTIMER+1                                       SUB02380
      IF (JSYM.GT.JMAX) GO TO 9                                          SUB02390
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      IF (K.GT.KEXTRA) MTIMER=0                                          SUB02400
      IF (M.GE.MLIMIT) GO TO 13                                          SUB02410
      IF (MTIMER.LE.MEXTRA) GO TO 2                                      SUB02420
13    IF (MEXTRA.EQ.0) GO TO 9                                           SUB02430
      TOTAL=0.                                                           SUB02440
      DO 14 MA=1,MEXTRA                                                  SUB02450
14    TOTAL=TOTAL+STM(MA+1)                                              SUB02470
      RMX=MEXTRA                                                         SUB02480
      TOTAL=TOTAL/RMX                                                    SUB02490
C   END OF M LOOP;  COMPUTE OUTPUT SOLN.                                 SUB02500
9     MOUT=M-1                                                           SUB02510
      IF (KOUT.EQ.0) KOUT=KMAX-1                                         SUB02520
      SOLN=TOTAL*EXPON*APP/2.                                            SUB02530
89    FORMAT ('   M=',I4,',  K=',I4,',  SUBTOTAL=',9E10.3)              SUB02540
88    FORMAT (2X,10E10.3)                                                SUB02550
      RETURN                                                             SUB02560
      END                                                                SUB02570
C000003                                                                  SUB02580
C


      SUBROUTINE BD (M,K,G,H,SCF,B,D,JJMX6)                              SUB02590
C                                                                        SUB02600
CC                                                                       SUB02610
C   BD EVALUATES THE FIRST (B) AND SECOND (D) COEFFICIENTS IN THE        SUB02620
C   INVERSION EQUATION, FOR A PARTICULAR SET OF INDEXES M & K.           SUB02630
C   BD MAKES USE OF THE HERMITE POLYNOMIAL ARRAY GENERATED BY            SUB02640
C   SUBROUTINE FIELD AS M & K ADVANCE.                                   SUB02650
C                                                                        SUB02660
      COMMON IMAX,JMAX,IIMX,JJMX,IJMX,ALPHA,SIZE,EPS,MODE,BOX,SD,IX,Z    SUB02670
      COMMON /SYM/ ISYM,JSYM,MSYM,FCU,IMS,JMS,QSYM                       SUB02680
      DIMENSION G(IJMX),SCF(JJMX6),H(IIMX)                               SUB02690
      PIE=3.141592653589793                                              SUB02700
      B=0.                                                               SUB02710
      D=0.                                                               SUB02720
      RM=M                                                               SUB02730
      RK=K                                                               SUB02740
      RJMAX=JMAX                                                         SUB02750
      JJMX4=4*JJMX                                                       SUB02760
      DXI=2.*PIE/FCU                                                     SUB02770
 200  FORMAT(1X,I10,/)
      IF (JSYM.LE.0) GO TO 4                                             SUB02780
      IF (M.NE.0) GO TO 2                                                SUB02790
      S=DXI                                                              SUB02800
      DO 1 J=1,JMAX                                                      SUB02810
      DO 1 I=1,IMAX                                                      SUB02820
      II=I+1                                                             SUB02830
      IJ=IMAX*(J-1)+I                                                    SUB02840
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      DH=H(II)-H(I)                                                     SUB02850
1     B=B+G(IJ)*S*DH                                                    SUB02860
      B=B*QSYM/2.                                                       SUB02870
      RETURN                                                            SUB02880
2     DO 3 J=1,JMAX                                                     SUB02890
      JS=J+JJMX4                                                        SUB02900
      S=SCF(JS)/RM                                                      SUB02910
      DO 3 I=1,IMAX                                                     SUB02920
      II=I+1                                                            SUB02930
      IJ=IMAX*(J-1)+I                                                   SUB02940
      DH=H(II)-H(I)                                                     SUB02950
3     B=B+G(IJ)*S*DH                                                    SUB02960
      B=B*QSYM                                                          SUB02970
      RETURN                                                            SUB02980
4     IF (M.NE.0) GO TO 6                                               SUB02990
      S=DXI                                                             SUB03000
      DO 5 J=1,JMAX                                                     SUB03010
      DO 5 I=1,IMAX                                                     SUB03020
      II=I+1                                                            SUB03030
      IJ=IMAX*(J-1)+I                                                   SUB03040
      DH=H(II)-H(I)                                                     SUB03050
5     B=B+G(IJ)*S*DH                                                    SUB03060
      B=B/2.                                                            SUB03070
      RETURN                                                            SUB03080
6     DO 7 J=1,JMAX                                                     SUB03090
      JS=J+JJMX4                                                        SUB03100
      J2=JS+JJMX                                                        SUB03110
      S=SCF(JS)/RM                                                      SUB03120
      C=SCF(J2)/RM                                                      SUB03130
      DO 7 I=1,IMAX                                                     SUB03140
      II=I+1
      IJ=IMAX*(J-1)+I                                                   SUB03160
      DH=H(II)-H(I)                                                     SUB03170
      B=B+G(IJ)*S*DH                                                    SUB03180
66    FORMAT ('  BD:',4I5,10F6.2)                                       SUB03190
7     D=D-G(IJ)*C*DH                                                    SUB03200
      RETURN                                                            SUB03210
      END                                                               SUB03220
C000004                                                                 SUB03230
C                                                                       SUB03240


      SUBROUTINE FIELD2 (RS,SIG,SOLN,G,H,SCF,DGN)                       SUB03250
C                                                                       SUB03260
C   FIELD2 COMPUTES THE SAME INVERSION AS SUBROUTINE FIELD, EXCEPT THAT SUB03270
C   THE COEFFICIENTS B AND D ARE COMPUTED INDIVIDUALLY AS USED BY       SUB03280
C   CALLING BD.  DISK STORAGE IS NOT REQUIRED, BUT COMPUTING TIME IS    SUB03290
C   MUCH GREATER.  FIELD2 IS UTILIZED BY SPECIFYING A NEGATIVE MODE ON  SUB03300
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C    THE INPUT PARAMETER.                                                    SUB03310
C    FIELD EVALUATES THE VALUE OF THE FIELD FUNCTION AT A PARTICULAR         SUB03320
C    POINT DESIGNATED IN CYLINDRICAL COORDINATES, BY USING THE INVERSION     SUB03330
C    EQUATION OF MALDONADO, ET AL.  FIELD CALLS SUBROUTINES BD & GARRAY.     SUB03340
C                                                                            SUB03350
      COMMON IMAX,JMAX,IIMX,JJMX,IJMX,ALPHA,SIZE,EPS,MODE,BOX,SD,IX,Z        SUB03360
      COMMON /TAB/ INDEX,KEXTRA,MEXTRA,KLIMIT,MLIMIT,KOUT,MOUT              SUB03370
      COMMON /SYM/ ISYM,JSYM,MSYM,FCU,IMS,JMS,QSYM                          SUB03380
      DIMENSION G(IJMX),H(IIMX,5),SCF(JJMX,6)                               SUB03390
C    INITIALIZE THE VALUES:                                                  SUB03400
      INDEX=0                                                               SUB03410
      MTIMER=0                                                              SUB03420
      KOUT=0                                                                SUB03430
      MOUT=0                                                                SUB03440
      MMAX=0                                                                SUB03450
      KMAX=0                                                                SUB03460
      TOTAL=0.                                                              SUB03470
      JJMX6=JJMX*6                                                          SUB03480
      IIMX2=(IIMX+1)/2                                                      SUB03490
      AR=ALPHA*RS                                                           SUB03500
      ARG=AR**2                                                             SUB03510
      EXPON=EXP(-ARG)                                                       SUB03520
      PIE=3.141592653589793                                                 SUB03530
      APP=ALPHA/PIE/PIE                                                     SUB03540
      M=0                                                                   SUB03550
      RM=M                                                                  SUB03560
      RIMAX=IMAX                                                            SUB03570
      DX=2./RIMAX                                                           SUB03580
      RJMAX=JMAX                                                            SUB03590
      DXI=2.*PIE/FCU                                                        SUB03600
C    INITIALIZE THE MODIFIED HERMITE POLYNOMIAL ARRAY;  VECTORS:             SUB03610
C    (1)=H1, (2)=H2, (3)=ALPHA*X(I), (4)=HM+2 STORED, (5)=HM+1 STORED       SUB03620
      DO 1 II=1,IIMX2                                                       SUB03630
      RII=II                                                                SUB03640
      IIM=IIMX-II+1                                                         SUB03650
      H(II,3)=ALPHA*(RII*DX-DX-1.)                                          SUB03660
      H(IIM,3)=-H(II,3)                                                     SUB03670
      H(II,1)=2.*H(II,3)                                                    SUB03680
      H(II,2)=(H(II,3)*H(II,1)-1.)/3.                                       SUB03690
      H(IIM,1)=-H(II,1)                                                     SUB03700
      H(IIM,2)=H(II,2)                                                      SUB03710
      H(II,5)=H(II,2)                                                       SUB03720
      H(IIM,5)=H(IIM,2)                                                     SUB03730
      H(II,4)=H(II,1)                                                       SUB03740
1     H(IIM,4)=H(IIM,1)                                                     SUB03750
      SIGN=1.                                                               SUB03760
      FM=1.                                                                 SUB03770
C    INITIATE THE SIN/COS ARRAY:                                             SUB03780
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      DO 11 J=1,JJMX                                                    SUB03790
      RJM=J-1                                                           SUB03800
      SCF(J,1)=0.                                                       SUB03810
      SCF(J,2)=1.                                                       SUB03820
      SCF(J,3)=SIN(RJM*DXI-PIE)                                         SUB03830
      SCF(J,4)=COS(RJM*DXI-PIE)                                         SUB03840
      SCF(J,5)=0.                                                       SUB03850
11    SCF(J,6)=0.                                                       SUB03860
      MS=0                                                              SUB03870
C   COMMENCE THE M LOOP:                                                SUB03880
2     SIGN=-SIGN                                                        SUB03890
      K=0                                                               SUB03900
      RK=K                                                              SUB03910
      ARM=1.                                                            SUB03920
      IF (M.NE.0) ARM=AR**M                                             SUB03930
      KTIMER=0                                                          SUB03940
      SIGNK=-1.                                                         SUB03950
C   COMPUTE THE K=0 & K=1 ORDERS OF LAGUERRE POLYNOMIAL FOR GIVEN M:    SUB03960
      PM=0.                                                             SUB03970
      P=SQRT(1./FM)                                                     SUB03980
      PP=(RM+1.-ARG)*SQRT(1./FM/(RM+1.))                                SUB03990
C   ADVANCE THE SIN/COS ARRAY FOR NEW M:                                SUB04000
      DO 12 J=1,JJMX                                                    SUB04010
      SCF(J,1)=SCF(J,1)*SCF(J,4)+SCF(J,2)*SCF(J,3)                      SUB04020
12    SCF(J,2)=SCF(J,2)*SCF(J,4)-SCF(J,1)*SCF(J,3)                      SUB04030
      DO 13 J=1,JMAX                                                    SUB04040
      SCF(J,5)=SCF(J+1,1)-SCF(J,1)                                      SUB04050
13    SCF(J,6)=SCF(J+1,2)-SCF(J,2)                                      SUB04060
C   TEST FOR SYMMETRY SKIPS:                                            SUB04070
      IF (MS.EQ.MSYM) MS=0                                              SUB04080
      TOTAL=0.                                                          SUB04090
      MS=MS+1                                                           SUB04100
      IF (MS.NE.1) GO TO 7                                              SUB04110
      RMS=RM*SIG                                                        SUB04120
      CMS=COS(RMS)                                                      SUB04130
      SMS=SIN(RMS)                                                      SUB04140
C   COMMENCE THE K LOOP:                                                SUB04150
3     INDEX=INDEX+1                                                     SUB04160
      SIGNK=-SIGNK                                                      SUB04170
C   CALL THE B & D COEFFICIENTS AND COMPUTE THE M,K SUMMATION TERM:     SUB04180
      CALL BD (M,K,G,H,SCF,B,D,JJMX6)                                   SUB04190
      IF (DGN.LE.-2.) WRITE (6,89) M,K,B,D                              SUB04200
      BRAKET=B                                                          SUB04210
      IF(RM.EQ.0.) GO TO 4                                              SUB04220
      BRAKET=B*CMS+D*SMS                                                SUB04230
4     ADD=SIGNK*BRAKET*P*ARM                                            SUB04240
      TOTAL=TOTAL+ADD                                                   SUB04250
C   ESTABLISH CHECK AS THE RELATIVE SIZE OF THE M,K TERM OF THE SERIES: SUB04260
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      CHECK=ABS(ADD)                                                    SUB04270
      IF (TOTAL.GT.EPS) CHECK=ABS(ADD/TOTAL)                            SUB04280
C     ADVANCE THE K INDEX:                                              SUB04290
      K=K+1                                                             SUB04300
      RK=K                                                              SUB04310
      ORDER=M+2*K+1                                                     SUB04320
C     GENERATE THE NEXT ORDER OF LAGUERRE POLYNOMIAL FOR NEW K:         SUB04330
      PM=P                                                              SUB04340
      P=PP                                                              SUB04350
      PP=P*(ORDER-ARG)-PM*SQRT(RK*(RM+RK))                              SUB04360
      PP=PP/SQRT((RK+1.)*(RM+RK+1.))                                    SUB04370
C     GENERATE THE NEXT ORDER OF THE SET OF HERMITE POLYNOMIALS FOR NEW K:SUB04380
      HA=SQRT(RK*(RK+RM))/ORDER                                         SUB04390
      HB=2.*SQRT((RK+1.)*(RM+RK+1.))/(ORDER+1.)/(ORDER+2.)              SUB04400
      DO 5 II=1,IIMX2                                                   SUB04410
      IIM=IIMX-II+1                                                     SUB04420
      H(II,1)=2.*(H(II,3)*H(II,2)-HA*H(II,1))                           SUB04430
      H(IIM,1)=SIGN*H(II,1)                                             SUB04440
5     H(II,2)=HB*(H(II,3)*H(II,1)-ORDER*H(II,2))                        SUB04450
C     SET K TIMER TO PROVIDE EXTRA K TERMS AFTER CHECK < EPS:           SUB04460
      KTIMER=KTIMER+1                                                   SUB04470
      IF (K.GE.KLIMIT) GO TO 6                                          SUB04480
      IF (CHECK.GE.EPS) KTIMER=0                                        SUB04490
      IF (KTIMER.LE.KEXTRA) GO TO 3                                     SUB04500
      GO TO 7                                                           SUB04510
C     END OF K LOOP;  ADVANCE M AND COMPUTE NEW TOTAL:                  SUB04520
6     KOUT=KOUT+1                                                       SUB04530
7     M=M+1                                                             SUB04540
      IF (K.GT.KMAX) KMAX=K                                             SUB04550
      RM=M                                                              SUB04560
      FM=FM*RM                                                          SUB04570
C     REGENERATE THE HERMITE ARRAY FOR NEW M, K=0:                      SUB04580
      DO 8 II=1,IIMX2                                                   SUB04590
      IIM=IIMX-II+1                                                     SUB04600
      H(II,2)=H(II,4)*SQRT(RM)/(RM+1.)                                  SUB04610
      H(II,1)=H(II,5)*(RM+2.)                                           SUB04620
      H(IIM,1)=-SIGN*H(II,1)                                            SUB04630
      H(II,2)=2.*SQRT(RM+1.)*(H(II,3)*H(II,1)-(RM+1.)*H(II,2))          SUB04640
      H(II,2)=H(II,2)/(RM+2.)/(RM+3.)                                   SUB04650
      H(II,4)=H(II,1)                                                   SUB04660
8     H(II,5)=H(II,2)                                                   SUB04670
C     SET M TIMER FOR EXTRA M TERMS:                                    SUB04680
      IF (MS.EQ.1) MTIMER=MTIMER+1                                      SUB04690
      IF (JSYM.GT.JMAX) GO TO 9                                         SUB04700
      IF (K.GT.KEXTRA) MTIMER=0                                         SUB04710
      IF (M.GE.MLIMIT) GO TO 9                                          SUB04720
      IF (MTIMER.LE.MEXTRA) GO TO 2                                     SUB04730
C     END OF M LOOP;  COMPUTE OUTPUT SOLN.                              SUB04740
```

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9     MOUT=M-1                                                                  SUB04750
      IF (KOUT.EQ.0) KOUT=KMAX-1                                                SUB04760
      SOLN=TOTAL*EXPON*APP/2.                                                   SUB04770
89    FORMAT ('   M=',I4,',   K=',I4,',   B=',E10.4,',   D=',E10.4)           SUB04780
      RETURN                                                                    SUB04790
      END                                                                       SUB04800
                                                                                SUB04810
                                                                                SUB04820
C000005                                                                         SUB00010
C                                                                               SUB00020

      SUBROUTINE GARRAY (G,GA,NOF,DGN,NUMB,XO,YO,PHISYM)                        SUB00030
C                                                                               SUB00040
CCC   GARRAY FILLS THE DATA ARRAY OVER AN ORTHOGONAL AREA WITH                  SUB00050
CCC   THE REGULAR DATA OBTAINED BY THE METHOD CORRESPONDING TO THE              SUB00060
CCC   PARTICULAR MODE:                                                         SUB00070
CCC                                                                             SUB00080
CCC        MODE 1  -  DATA OBTAINED BY SAMPLING A KNOWN FUNCTION SUPPLIED       SUB00090
CCC                   IN SUBROUTINE FUNCT AND SAMPLED IN SUBROUTINE GOLF.       SUB00100
CCC                                                                             SUB00110
CCC        MODE 2  -  DATA OBTAINED BY GENERATING A REGULAR ARRAY FROM          SUB00120
CCC                   IRREGULAR EXPERIMENTAL INPUT DATA READ IN.  CALLS         SUB00130
CCC                   SUBROUTINE SHEET. (EXPERIMENTAL DATA MAY                  SUB00140
CCC                   BE SIMULATED, SEE 'SHEET')                                SUB00150
CCC                                                                             SUB00160
CCC        MODE 3  -  UTILIZES RAW DATA TAKEN AT THE PROPER INTERVAL,           SUB00170
CCC                   OR PREVIOUSLY GENERATED, AND READ DIRECTLY INTO THE       SUB00180
CCC                   GARRAY.  CALLS SUBROUTINE READ.                           SUB00190
C                                                                               SUB00200
      COMMON IMAX,JMAX,IIMX,JJMX,IJMX,ALPHA,SIZE,EPS,MODE,BOX,SD,IX,Z           SUB00210
      COMMON /SYM/ ISYM,JSYM,MSYM,FCU,IMS,JMS,QSYM                              SUB00220
      COMMON /IO/ CMS,IN1,IN2,IN4                                               SUB00230
      DIMENSION G(IMAX,JMAX),GA(IMAX,JMAX)                                      SUB00240
      PIE=3.141592653589793                                                     SUB00250
      HS=SIZE/2.                                                                SUB00260
      IF (MODE.GT.3) MODE=1                                                     SUB00270
      RIMX=IMAX                                                                 SUB00280
      RJMX=JMAX                                                                 SUB00290
      DELR=SIZE/RIMX                                                            SUB00300
      DELXI=2.*PIE/FCU                                                          SUB00310
      IF (MODE.GT.1) GO TO 2                                                    SUB00320
      DO 1 J=1,JMS                                                              SUB00330
      RJ=J                                                                      SUB00340
      XI=(RJ-.5)*DELXI-PIE                                                      SUB00350
      J2=J+2*(JMS-J)                                                            SUB00360
      J3=J+JMAX/2                                                               SUB00370
      J4=J2+JMAX/2                                                              SUB00380
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      DO 1 I=1,IMS                                                      SUB00390
      RI=I                                                              SUB00400
      II=IMAX+1-I                                                       SUB00410
      R=(RI-.5)*DELR-HS                                                 SUB00420
      CALL GOLF (R,XI,GIJ,NOF,DGN,NUMB)                                 SUB00430
      G(I,J)=GIJ                                                        SUB00440
      IF (ISYM.EQ.2) G(II,J)=GIJ                                        SUB00450
      IF (ISYM.EQ.2) GO TO 1                                            SUB00460
      G(II,J3)=GIJ                                                      SUB00470
      IF (JSYM.EQ.0) GO TO 1                                            SUB00480
      G(I,J2)=GIJ                                                       SUB00490
      G(II,J4)=GIJ                                                      SUB00500
1     CONTINUE                                                          SUB00510
      GO TO 4                                                           SUB00520
2     IF (MODE.GT.2) GO TO 3                                            SUB00530
      CALL SHEET (G,GA,XO,YO,PHISYM,NOF)                                SUB00540
      GO TO 4                                                           SUB00550
3     CALL READ (Z,XO,YO,PHISYM,NOF,IMAX,JMAX,G)                        SUB00560
4     IF (DGN.GE.2) WRITE (6,39)                                        SUB00570
      RETURN                                                            SUB00580
39    FORMAT ('   GARRAY RETURNS')                                      SUB00590
      END                                                               SUB00600
C000006                                                                 SUB00610
C                                                                       SUB00620


      SUBROUTINE GOLF (R,XI,GIJ,NOF,DGN,NUMB)                           SUB00630
C                                                                       SUB00640
C   GOLF COMPUTES THE FUNCTION G(R,XI) FOR A PARTICULAR LINE OF SIGHT   SUB00650
C   FROM A KNOWN FUNCTION CONTAINED IN SUBROUTINE FUNCT.                SUB00660
C                                                                       SUB00670
      COMMON IMAX,JMAX,IIMX,JJMX,IJMX,ALPHA,SIZE,EPS,MODE,BOX,SD,IX,Z   SUB00680
      ZERO=0.                                                           SUB00690
      LMAX=IMAX*3                                                       SUB00700
      RLMAX=LMAX                                                        SUB00710
      DELXP=SIZE/RLMAX                                                  SUB00720
      SXI=SIN(XI)                                                       SUB00730
      CXI=COS(XI)                                                       SUB00740
      DELXS=DELXP*CXI                                                   SUB00750
      DELYS=DELXP*SXI                                                   SUB00760
      XP=DELXP*.5-SIZE/2.                                               SUB00770
      XS=XP*CXI-R*SXI                                                   SUB00780
      YS=XP*SXI+R*CXI                                                   SUB00790
      GIJ=0.                                                            SUB00800
      DO 1 L=1,LMAX                                                     SUB00810
      RL=L                                                              SUB00820
      CALL FUNCT(XS,YS,F,NOF,DGN,NUMB)                                  SUB00830
      GIJ=GIJ+F                                                         SUB00840
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      XS=XS+DELXS                                                       SUB00850
1     YS=YS+DELYS                                                       SUB00860
      IF (GIJ.NE.0.) GIJ=GIJ*DELXP*BOX                                  SUB00870
      IF ((SD.EQ.0.).OR.(NUMB.EQ.1)) GO TO 2                            SUB00880
      IF (DGN.GE.3) WRITE (6,28) IX                                     SUB00890
      CALL GAUSS (IX,SD,ZERO,RV)                                        SUB00900
      GIJ=GIJ+RV                                                        SUB00910
2     IF (DGN.GE.3) WRITE (6,29) R,XI,GIJ                               SUB00920
      RETURN                                                            SUB00930
29    FORMAT ('   R=',F8.3,',   XI=',F8.3,',   GIJ=',F8.3)            SUB00940
28    FORMAT ('   GAUSS, IX=',I8)                                       SUB00950
      END                                                               SUB00960
C000007                                                                 SUB00970
C                                                                       SUB00980

      SUBROUTINE FUNCT (XS,YS,F,NOF,DGN,NUMB)                           SUB00990
C                                                                       SUB01000

CP67USERID  1095BOXJ
C
C   FUNCT EVALUATES AS INPUT FUNCTION AT POSITION (X,Y) IN THE TEST     SUB01020
C   SECTION COORDINATE SYSTEM.  NOF IDENTIFIES THE EQUATION USED.       SUB01030
C                                                                       SUB01040
      COMMON IMAX,JMAX,IIMX,JJMX,IJMX,ALPHA,SIZE,EPS,MODE,BOX,SD,IX,Z   SUB01050
      COMMON /EQPARA/ A,B,C,D,E,P,Q,S,T,U,V,W,RO,RA,NO,NA,N1,N2         SUB01060
      DIMENSION RO(101),RA(101)                                         SUB01070
      AA=A                                                              SUB01080
      BB=B                                                              SUB01090
      CC=C                                                              SUB01100
      DD=D                                                              SUB01110
      EE=E                                                              SUB01120
      PP=P                                                              SUB01130
      IF (NUMB.LE.1) GO TO 50                                           SUB01140
      AA=S                                                              SUB01150
      BB=T                                                              SUB01160
      CC=U                                                              SUB01170
      DD=V                                                              SUB01180
      EE=W                                                              SUB01190
      PP=Q                                                              SUB01200
50    PIE=3.141592653589793                                             SUB01210
      HS=SIZE/2.                                                        SUB01220
      R=SQRT(XS**2+YS**2)/HS                                            SUB01230
      F=0.                                                              SUB01240
      IF (R.GT.1.) GO TO 11                                             SUB01250
      IF (NOF.LE.0) GO TO 11                                            SUB01260
C                                                                       SUB01270
C   1.  AXISYMMETRIC GAUSSIAN:                                          SUB01280
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```
1       IF (NOF.GT.1) GO TO 2                                                  SUB01290
        F=AA*EXP(-1.*(R*HS/BB)**2)                                             SUB01300
        GO TO 11                                                               SUB01310
C                                                                              SUB01320
C     2.  ADJUSTABLE RECTANGULAR STEP FUNCTION:                                SUB01330
2       IF (NOF.GT.2) GO TO 3                                                  SUB01340
        F=PP                                                                   SUB01350
        IF ((ABS(XS-DD).LE.BB).AND.(ABS(YS-EE).LE.CC)) F=AA                    SUB01360
        GO TO 11                                                               SUB01370
C                                                                              SUB01380
C     3.  DISPLACABLE ELLIPTICAL GAUSSIAN:                                     SUB01390
3       IF (NOF.GT.3) GO TO 4                                                  SUB01400
        F=AA*EXP(-1.*(((XS-DD)/BB)**2+((YS-EE)/CC)**2))                        SUB01410
        GO TO 11                                                               SUB01420
C                                                                              SUB01430
C     4.  CONSTANT:                                                            SUB01440
4       IF (NOF.GT.4) GO TO 5                                                  SUB01450
        F=AA                                                                   SUB01460
        GO TO 11                                                               SUB01470
C                                                                              SUB01480
C     5.  ADJUSTABLE AND DISPLACABLE ELLIPTIC RAMP FUNCTION:                   SUB01490
5       IF (NOF.GT.5) GO TO 6                                                  SUB01500
        RBC=SQRT(((XS-DD)/BB)**2+((YS-EE)/CC)**2)                              SUB01510
        F=0.                                                                   SUB01520
        IF (RBC.LT.1.) F=AA*((1.-RBC)**PP)                                     SUB01530
        GO TO 11                                                               SUB01540
C                                                                              SUB01550
C     6.  DISPLACABLE ELLIPTIC STEP FUNCTION:                                  SUB01560
6       IF (NOF.GT.6) GO TO 7                                                  SUB01570
        RBC=SQRT(((XS-DD)/BB)**2+((YS-EE)/CC)**2)                              SUB01580
        F=0.                                                                   SUB01590
        IF (RBC.LT.1.) F=AA                                                    SUB01600
        GO TO 11                                                               SUB01610
C                                                                              SUB01620
C     7.  CIRCULAR COSINE-SQUARED FUNCTION OF BB MAXIMA:                       SUB01630
7       IF (NOF.GT.7) GO TO 8                                                  SUB01640
        F=AA*COS((2.*BB-1.)*PIE*R/2.)**2                                       SUB01650
        GO TO 11                                                               SUB01660
C                                                                              SUB01670
C     8.  NUMERICAL FUNCTION:  REQUIRES AN INPUT ARRAY READ IN BY              SUB01680
C         SUBROUTINE FREAD; N FOLLOWED BY N POINT VALUES. (101 MAX)            SUB01690
C         A CONSTANT VALUE AA IS ADDED TO THE FUNCTION.                        SUB01700
8       IF (NOF.GT.8) GO TO 9                                                  SUB01710
        IF (NUMB.LE.1) N=NO                                                    SUB01720
        IF (NUMB.GT.1) N=NA                                                    SUB01730
        NM=N-1                                                                 SUB01740
        NMM=N-2                                                                SUB01750
        RN=N                                                                   SUB01760
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      RI=R*(RN-1.)+1.                                                    SUB01770
      IR=INT(RI)                                                         SUB01780
      RIR=FLOAT(IR)                                                      SUB01790
      DI=RI-RIR                                                          SUB01800
      IF (NUMB.LE.1) F=RO(IR)                                            SUB01810
      IF (NUMB.GT.1) F=RA(IR)                                            SUB01820
      IF ((IR.NE.N).AND.(NUMB.LE.1)) F=F+DI*(RO(IR+1)-RO(IR))            SUB01830
      IF ((IR.NE.N).AND.(NUMB.GT.1)) F=F+DI*(RA(IR+1)-RA(IR))            SUB01840
      F=F*AA+BB                                                          SUB01850
      GO TO 11                                                           SUB01860
C                                                                        SUB01870
C   9.  SPECIAL FUNCTION:  MAY BE WRITTEN FOR THE OCCASION AND           SUB01880
C       INSERTED IN SUBROUTINE SPFUN                                     SUB01890
9     IF (NOF.GT.9) GO TO 10                                             SUB01900
      CALL SPFUN (XS,YS,F)                                               SUB01910
      GO TO 11                                                           SUB01920
C                                                                        SUB01930
C   EQUATIONS NO. 10 AND BEYOND ARE SET TO ZERO.                         SUB01940
10    F=0.                                                               SUB01950
C                                                                        SUB01960
C                                                                        SUB01970
11    IF (DGN.GE.4) WRITE  (6,99) XS,YS,F                                SUB01980
99    FORMAT ('   XS=',F8.3,',   YS=',F8.3,',   F=',F8.3)               SUB01990
      RETURN                                                             SUB02000
      END                                                                SUB02010
C000008                                                                  SUB02020
C                                                                        SUB02030


      SUBROUTINE SPFUN (XS,YS,F)                                         SUB02040
C                                                                        SUB02050
C   SPFUN IS A SPECIAL ROUTINE FOR EQ'N NO. 9.  ANY FUNCTION MAY BE      SUB02060
C   ENTERED.                                                             SUB02070
C                                                                        SUB02080
      COMMON /EQPARA/ A,B,C,D,E,P,Q,S,T,U,V,W,RO,RA,NO,NA,N1,N2          SUB02090
      DIMENSION RO(101),RA(101)                                          SUB02100
      F=0.                                                               SUB02110
      IF ((ABS(XS).LE.B).AND.(ABS(YS).LE.C))  F=A                        SUB02120
      RETURN                                                             SUB02130
      END                                                                SUB02140
C000009                                                                  SUB02150
C                                                                        SUB02160
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      SUBROUTINE SHEET (G,D,XO,YO,PHISYM,NOF)                          SUB02170
                                                                       SUB02180
C                                                                      SUB02190
CC    SHEET READS IRREGULARLY SPACED VALUES OF THE LINE INTEGRAL, AS   SUB02200
CC    OBTAINED FROM HOLOGRAPHIC INTERFEROGRAMS.  THE INTEGRAL LINES MAY BESUB02210
CC    DEFINED EITHER BY GRID INTERCEPT POSITIONS, OR BY ANGLE AND RADIUSSUB02220
CC    ABOUT THE CENTER OF THE LABORATORY COORDINATE SYSTEM CENTER.  LINESSUB02230
CC    MUST BE ENTERED IN CONSECUTIVE ORDER FROM LOWEST (NEG.) TO HIGHESTSUB02240
CC    (POS.) RADIUS.  DATA MAY BE SIMULATED BY SPECIFYING NCODE=1,     SUB02250
CC    FOLLOWED BY APERATURE POSITIONS FOR A FUNCTION NUMBER IN 'SUBFUNCT'.SUB02260
C                                                                      SUB02270
      COMMON IMAX,JMAX,IIMX,JJMX,IJMX,ALPHA,SIZE,EPS,MODE,BOX,SD,IX,Z  SUB02280
      COMMON /SYM/ISYM,JSYM,MSYM,FCU,IMS,JMS,QSYM                      SUB02290
      COMMON /IO/ CMS,IN1,IN2,IN4                                      SUB02300
      DIMENSION G(IMAX,JMAX),D(IMAX,JMAX),XI(303),RR(303)              SUB02310
      DIMENSION XG(303),XD(303),YG(303),YD(303),XY(303)                SUB02320
      NAR=303                                                          SUB02330
      PIE=3.141592653589793                                            SUB02340
      MPIE=-PIE                                                        SUB02350
      TPIE=2.*PIE                                                      SUB02360
      MPIT=MPIE/2.                                                     SUB02370
      PIET=PIE/2.                                                      SUB02380
C     ZERO THE ARRAYS:                                                 SUB02390
      DO 1 J=1,JMAX                                                    SUB02400
      DO 1 I=1,IMAX                                                    SUB02410
      G(I,J)=0.                                                        SUB02420
1     D(I,J)=0.                                                        SUB02430
      DO 2 I=1,NAR                                                     SUB02440
      XG(I)=0.                                                         SUB02450
      XD(I)=0.                                                         SUB02460
      YG(I)=0.                                                         SUB02470
      YD(I)=0.                                                         SUB02480
      XY(I)=0.                                                         SUB02490
      XI(I)=0.                                                         SUB02500
2     RR(I)=0.                                                         SUB02510
C     READ THE BASIC DATA:                                             SUB02520
      IF (CMS.EQ.1.) REWIND 1                                          SUB02530
      READ (IN1,59) NOF,NCODE                                          SUB02540
      READ (IN1,58) Z,XO,YO,PHISYM,XMX,XMN,YMX,YMN                     SUB02550
      READ (IN1,59) JM                                                 SUB02560
      RIMX=IMAX                                                        SUB02570
      DR=SIZE/RIMX                                                     SUB02580
      R=(-DR-SIZE)/2.                                                  SUB02590
      RZO=SQRT(XO**2+YO**2)                                            SUB02600
      GAM=ATANM(YO,XO)                                                 SUB02610
      TP=3-ISYM                                                        SUB02620
      BT=JSYM                                                          SUB02630
      DAN=PIE*TP/BT                                                    SUB02640
      HS=SIZE/2.
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      MXY=1                                                                  SUB02650
      IF((XMX.NE.0.).OR.(XMN.NE.0.).OR.(YMX.NE.0.).OR.(YMN.NE.0.))MXY=0      SUB02660
      IF (MXY.EQ.1) GO TO 3                                                  SUB02670
      XMX=XO+HS                                                              SUB02680
      YMX=YO+HS                                                              SUB02690
      XMN=XO-HS                                                              SUB02700
      YMN=YO-HS                                                              SUB02710
C     COMMENCE THE READ AND FILL LOOP:                                       SUB02720
3     DO 12 J=1,JM                                                           SUB02730
      READ (IN1,59)IM                                                        SUB02740
      MN=0                                                                   SUB02750
      XH=0.                                                                  SUB02760
      YH=0.                                                                  SUB02770
C     READ THE LINES, DETERMINE CODE, CALCULATE RADIUS & ANGLE FOR CODE 1:   SUB02780
      DO 5 I=1,IM                                                            SUB02790
      IF(NCODE.LE.0)READ(IN1,58) XD(I),YD(I),XG(I),YG(I),D(I,J),RR(I),       SUB02800
     1XI(I),XY(I)                                                            SUB02810
      IF(NCODE.GE.1)CALL SIM(XD(I),YD(I),XG(I),YG(I),RR(I),XI(I),            SUB02820
     1XY(I),XO,YO,PHISYM,XMX,XMN,YMX,YMN,NOF,I,IM)                           SUB02830
      IF (XY(I).EQ.3.) GO TO 5                                               SUB02840
      IF((XD(I).NE.0.).OR.(YD(I).NE.0.)) XY(I)=1.                            SUB02850
      IF((XG(I).NE.0.).OR.(YG(I).NE.0.)) XY(I)=1.                            SUB02860
      IF(((RR(I).NE.0.).OR.(XI(I).NE.0.)).AND.(XY(I).EQ.0.)) XY(I)=2.        SUB02870
      IF((XY(I).EQ.0.).AND.(D(I,J).NE.0.)) XY(I)=2.                          SUB02880
      IF (XY(I).NE.1.) GO TO 4                                               SUB02890
      DEN=SQRT((XG(I)-XD(I))**2+(YG(I)-YD(I))**2)                            SUB02900
      IF (DEN.EQ.0.) XY(I)=4.                                                SUB02910
      IF (XY(I).EQ.4.) GO TO 4                                               SUB02920
      RR(I)=((XO-XD(I))*(YG(I)-YD(I))-(XG(I)-XD(I))*(YO-YD(I)))/DEN          SUB02930
      XI(I)=ATANM((YG(I)-YD(I)),(XG(I)-XD(I)))                               SUB02940
      XIM=XI(I)                                                              SUB02950
4     IF (XY(I).EQ.2.) XIM=XI(I)                                             SUB02960
      XIN=XIM                                                                SUB02970
      IF(XY(I).EQ.2.) RR(I)=RR(I)+RZO*SIN(GAM-XI(I))                         SUB02980
5     CONTINUE                                                               SUB02990
C     COMPUTE MAX AND MIN ANGLE INDEXES FOR APERATURE POSITION LOCATION:     SUB03000
      DO 6 I=1,IM                                                            SUB03010
      IF((XY(I).NE.1.).OR.(XY(I).NE.2.)) GO TO 6                             SUB03020
      IF (XI(I).GT.XIM) XIM=XI(I)                                            SUB03030
      IF (XI(I).GT.XIM) IMT=I                                                SUB03040
      IF (XI(I).LT.XIN) XIN=XI(I)                                            SUB03050
      IF (XI(I).LE.XIN) INT=I                                                SUB03060
6     CONTINUE                                                               SUB03070
C     DETERMINE APERATURE LOCATION:                                          SUB03080
      LPR=0                                                                  SUB03090
      XID=XI(IMT)-XI(INT)                                                    SUB03100
      IF(ABS(XID).LT..00001) LPR=1                                           SUB03110
      XIH=(XI(IMT)+XI(INT))/2.                                               SUB03120
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      RRH=10000.                                                        SUB03130
      XH=RRH*COS(XIH)                                                   SUB03140
      YH=RRH*SIN(XIH)                                                   SUB03150
      IF (LPR.EQ.1) GO TO 7                                             SUB03160
      YTX=-RR(IMT)*SIN(XI(IMT))-YO                                      SUB03170
      YTN=-RR(INT)*SIN(XI(INT))-YO                                      SUB03180
      XTX=RR(IMT)*COS(XI(IMT))-XO                                       SUB03190
      XTN=RR(INT)*COS(XI(INT))-XO                                       SUB03200
      UA=TAN(XI(IMT))                                                   SUB03210
      UC=TAN(XI(INT))                                                   SUB03220
      UB=YTX-UA*XTX                                                     SUB03230
      UD=YTN-UC*XTN                                                     SUB03240
      XH=(UD-UB)/(UA-UC)                                                SUB03250
      YH=XH*UA+UB                                                       SUB03260
      RRH=SQRT((XH-XO)**2+(YH-YO)**2)                                   SUB03270
      XIH=ATANM((YH-YO),(XH-XO))                                        SUB03280
7     CONTINUE                                                          SUB03290
C   FILL THE ANGLE AND RADIUS FOR ANY CODE 3 OR 4 LINES:                SUB03300
      DO 9 I=1,IM                                                       SUB03310
      IF(XY(I).NE.3.) GO TO 8                                           SUB03320
      BAS=SQRT(RRH**2-RR(I)**2)                                         SUB03330
      XI(I)=XIH-ATANM(RR(I),BAS)                                        SUB03340
      GO TO 9                                                           SUB03350
8     XI(I)= ATANM((YH-YD(I)),(XH-XD(I)))                               SUB03360
      RR(I)=RRH*SIN(XI(I)-XIH)                                          SUB03370
9     CONTINUE                                                          SUB03380
C   ANGLES AND RADII ARE NOW FILLED FOR ALL POINTS IN THIS LINE.        SUB03390
C   VACATE THE SET OF VECTORS TO BE USED AS TEMPORARY STORAGE:          SUB03400
      DO 10 I=1,IM                                                      SUB03410
      XD(I)=0.                                                          SUB03420
      YD(I)=0.                                                          SUB03430
      XG(I)=0.                                                          SUB03440
      YG(I)=RR(I)                                                       SUB03450
      XY(I)=D(I,J)                                                      SUB03460
      RR(I)=0.                                                          SUB03470
      D(I,J)=0.                                                         SUB03480
10    XI(I)=0.                                                          SUB03490
C   CONVERT THE LINE TO REGULAR RADII USING INTERPOLATION:              SUB03500
      RR(1)=R+DR                                                        SUB03510
      CALL SPLINE(YG,XY,IM,RR(1),D(1,J))                                SUB03520
      DO 11 I=2,IMAX                                                    SUB03530
      RI=I                                                              SUB03540
      RR(I)=R+DR*RI                                                     SUB03550
11    CALL SPLINN(YG,XY,IM,RR(I),D(I,J))                                SUB03560
C   GENERATE THE VECTOR OF ANGLES FOR THIS COLUMN AND STORE IN G ARRAY: SUB03570
      DO 12 I=1,IMAX                                                    SUB03580
      BAS=SQRT(RRH**2-RR(I)**2)                                         SUB03590
      G(I,J)=XIH-ATANM(RR(I),BAS)                                       SUB03600
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      YG(I)=0.                                                          SUB03610
      D(I,J)=XY(I)                                                      SUB03620
12    XY(I)=0.                                                          SUB03630
C   COLUMNS ARE NOW ALL REGULARLY FILLED.                               SUB03640
C   NEXT,INTERPOLATE EACH ROW REGULARLY OVER THE ANGLES.                SUB03650
      DO 23 I=1,IMAX                                                    SUB03660
C   EXPAND THE DATA TO 2 SETS TO ESTABLISH SMOOTH INTERPOLATION.        SUB03670
      JM3=3*JM                                                          SUB03680
      II=IMAX+1-I                                                       SUB03690
      IF (JSYM.NE.0) GO TO 14                                           SUB03700
      DO 13 J=1,JMS                                                     SUB03710
      J2=J+JMS                                                          SUB03720
      J3=J2+JMS                                                         SUB03730
      XD(J)=D(I,J)                                                      SUB03740
      XD(J2)=D(II,J)                                                    SUB03750
      XD(J3)=D(I,J)                                                     SUB03760
      XG(J)=G(I,J)-TPIE-PHISYM                                          SUB03770
      XG(J2)=G(II,J)-PIE-PHISYM                                         SUB03780
13    XG(J3)=G(I,J)-PHISYM                                              SUB03790
      GO TO 16                                                          SUB03800
14    DO 15 J=1,JMS                                                     SUB03810
      J1=JMS+1-J                                                        SUB03820
      J2=JMS+J                                                          SUB03830
      J3=JM3+1-J                                                        SUB03840
      XD(J1)=D(I,J)                                                     SUB03850
      XD(J2)=D(II,J)                                                    SUB03860
      XD(J3)=D(II,J)                                                    SUB03870
      XG(J1)=G(I,J)-2.*(G(I,J)-PHISYM)-PIE-PHISYM                       SUB03880
      XG(J2)=G(II,J)-PIE-PHISYM                                         SUB03890
15    XG(J3)=G(II,J)+2.*(DAN+PHISYM-G(II,J))-PIE-PHISYM                 SUB03900
16    CONTINUE                                                          SUB03910
      JM2=2*JMS                                                         SUB03920
      JP=JMS/2                                                          SUB03930
      DO 17 J=1,JM2                                                     SUB03940
      XD(J)=XD(J+JP)                                                    SUB03950
17    XG(J)=XG(J+JP)                                                    SUB03960
      JJS=JM2+1                                                         SUB03970
      DO 18 J=JJS,JM3                                                   SUB03980
      XD(J)=0.                                                          SUB03990
18    XG(J)=0.                                                          SUB04000
C   FIND THE SMALLEST ANGLE                                             SUB04010
      XY(1)=1.                                                          SUB04020
      SA=XG(1)                                                          SUB04030
      DO 19 J=1,JM2                                                     SUB04040
      IF (XG(J).GE.SA) GO TO 19                                         SUB04050
      SA=XG(J)                                                          SUB04060
      XY(1)=J                                                           SUB04070
19    CONTINUE                                                          SUB04080
```

```
C    FIND THA MAX ANGLE IN THE ROW:                                    SUB04090
       XY(JM2)=JM2                                                     SUB04100
       SB=XG(JM2)                                                      SUB04110
       DO 20 J=1,JM2                                                   SUB04120
       IF (XG(J).LE.SB) GO TO 20                                       SUB04130
       SB=XG(J)                                                        SUB04140
       XY(JM2)=J                                                       SUB04150
20     CONTINUE                                                        SUB04160
C    DETERMINE THE ORDER OF INCREASING ANGLE IN THE ROW                SUB04170
       SB=XG(JM2)                                                      SUB04180
       JJ=2                                                            SUB04190
21     JSA=XY(JJ-1)                                                    SUB04200
       SA=XG(JJ-1)                                                     SUB04210
       JTS=0                                                           SUB04220
       DO 22 J=1,JM2                                                   SUB04230
       IF (XG(J).LE.SA) GO TO 22                                       SUB04240
       IF (XG(J).GT.SB) GO TO 22                                       SUB04250
       SB=XG(J)                                                        SUB04260
       XY(JJ)=J                                                        SUB04270
       JTS=1                                                           SUB04280
22     CONTINUE                                                        SUB04290
       IF (JTS.EQ.0) JM2=JJ                                            SUB04300
       JJ=JJ+1                                                         SUB04310
       IF (JJ.LE.JM2) GO TO 21                                         SUB04320
       DO 23 J=1,JM2                                                   SUB04330
       JX=XY(J)                                                        SUB04340
23     YD(J)=XD(JX)                                                    SUB04350
C    INTERPOLATE:                                                      SUB04360
       DXI=2.*PIE/FCU                                                  SUB04370
       XI(J)=DXI/2.-PIE-PHISYM                                         SUB04380
       CALL SPLINE (XG,YD,JM2,XI(J),G(I,J))                            SUB04390
       DO 24 J=2,JMS                                                   SUB04400
       XI(J)=XI(J-1)+DXI                                               SUB04410
24     CALL SPLINN (XG,YD,JM2,XI(J),G(I,J))                            SUB04420
       DO 25 J=1,JMS                                                   SUB04430
       XIJ=XI(J)                                                       SUB04440
       XU=XMX                                                          SUB04450
       IF ((XIJ.GE.0.).AND.(XIJ.LT.PIE)) XU=XMN                        SUB04460
       YU=YMN                                                          SUB04470
       IF ((XIJ.GE.MPIT).AND.(XIJ.LT.PIT)) YU=YMX                      SUB04480
       XL=XMN                                                          SUB04490
       IF ((XIJ.GE.0.).AND.(XIJ.LT.PIE)) XL=XMX                        SUB04500
       YL=YMX                                                          SUB04510
       IF ((XIJ.GE.MPIT).AND.(XIJ.LT.PIT)) YL=YMN                      SUB04520
       SXIJ=SIN(XIJ)                                                   SUB04530
       CXIJ=COS(XIJ)                                                   SUB04540
       RMN=(XO-XL)*SXIJ-(YO-YL)*CXIJ                                   SUB04550
       RMX=(XO-XU)*SXIJ-(YO-YU)*CXIJ                                   SUB04560
```

```
      DO 25 I=1,IMAX                                                    SUB04570
      IF (RR(I).LT.RMN) G(I,J)=0.                                       SUB04580
      IF (RR(I).GT.RMX) G(I,J)=0.                                       SUB04590
25    CONTINUE                                                          SUB04600
C   EXPAND SYMMETRY SECTOR INTO AN ORTHOGONAL INTERVAL.                 SUB04610
      IF (ISYM.EQ.2) GO TO 27                                           SUB04620
      DO 26 J=1,JMS                                                     SUB04630
      J2=JMAX/2+1-J                                                     SUB04640
      J3=JMAX/2+J                                                       SUB04650
      J4=JMAX+1-J                                                       SUB04660
      DO 26 I=1,IMAX                                                    SUB04670
      II=IMAX+1-I                                                       SUB04680
      G(I,J2)=G(I,J)                                                    SUB04690
      G(II,J3)=G(I,J)                                                   SUB04700
26    G(II,J4)=G(I,J)                                                   SUB04710
      RETURN                                                            SUB04720
C   FOR EVEN SYMMETRY, AVERAGE THE GARRAY COLUMNS.                      SUB04730
27    IMS=(2*IMAX+1)/2                                                  SUB04740
      DO 28 J=1,JMAX                                                    SUB04750
      DO 28 I=1,IMS                                                     SUB04760
      II=IMAX+1-I                                                       
      GST=(G(I,J)+G(II,J))/2.                                           SUB04780
      G(I,J)=GST                                                        SUB04790
28    G(II,J)=GST                                                       SUB04800
      RETURN                                                            SUB04810
59    FORMAT (5I5)                                                      SUB04820
58    FORMAT (10F7.3)                                                   SUB04830
      END                                                               SUB04840
C000010                                                                 SUB04850
C                                                                       SUB04860


      FUNCTION ATANM(Y,X)                                               SUB04870
C                                                                       SUB04880
C   COMPUTES THE ARCTAN OF Y/X BETWEEN -P1 AND +P1.                     SUB04890
C                                                                       SUB04900
      PIE=3.141592653589793                                             SUB04910
      PI2=PIE/2.                                                        SUB04920
      ATANM=SIGN(PI2,Y)                                                 SUB04930
      IF(X.NE.0.) ATANM=ATAN(Y/X)                                       SUB04940
      IF(X.GE.0.) RETURN                                                SUB04950
      IF(Y.GE.0.) ATANM=PIE+ATANM                                       SUB04960
      IF(Y.LT.0.) ATANM=-PIE+ATANM                                      SUB04970
      RETURN                                                            SUB04980
      END                                                               SUB04990
C000011                                                                 SUB05000
C                                                                       SUB05010
```

```
      SUBROUTINE SIM (XD,YD,XG,YG,D,R,XI,XY,XO,YO,PS,XM,XN,YM,YN,I,IM,   SUB05020
     1DG,NF)                                                            SUB05030
C    SIM SIMULATES THE FRINGE NUMBER DATA ONE WOULD OBTAIN FROM THE     SUB05040
C    HOLOGRAPHIC INTERFEROGRAM PROCESS FOR A KNOWN FUNCTION AS          SUB05050
C    CONTAINED IN SUBROUTINE FUNCT.  THE GRID BOX DIMENSIONS MUST       SUB05060
C    EXCEED THE INVERSION CIRCLE SIZE, AND APERATURE POINTS SPECIFIED   SUB05070
C    MUST FALL BETWEEN XI=-40 DEGREES, AND XI=+130 DEGREES.             SUB05080
C                                                                       SUB05090
      COMMON IMAX,JMAX,IIMX,JJMX,IJMX,ALF,SIZ,EPS,MOD,BOX,SD,IX,Z       SUB05100
      COMMON /IO/ CMS,IN1,IN2,IN4                                       SUB05110
      READ (IN1,29) XH,YH                                               SUB05120
      ZER=0.                                                            SUB05130
      RIM=IM                                                            SUB05140
      RI=I                                                              SUB05150
      DX=(XM-XN)/(RIM-1.)                                               SUB05160
      DY=(YM-YN)/(RIM-1.)                                               SUB05170
      X1=XM-(RIM-1.)*DX                                                 SUB05180
      Y1=YM-(RIM-1.)*DY                                                 SUB05190
      XI1=ATANM (YH-Y1,XH-X1)-PS                                        SUB05200
      RRH=SQRT((XH-XO)**2+(YH-YO)**2)                                   SUB05210
      XIO=ATANM(YH-YO,XH-XO)-PS                                         SUB05220
      R1=RRH*SIN(XI1-XIO)                                               SUB05230
      IF ((I.GT.1).AND.(I.LT.IM)) GO TO 1                               SUB05240
      IF (ABS(R1).LT.SIZ/2.) R1=SIGN(SIZ/2.,R1)                         SUB05250
      XY=3.                                                             SUB05260
      GO TO 2                                                           SUB05270
1     CALL GOLF (R,XI,D,NR,ZER,ZER)                                     SUB05280
      XD=XN                                                             SUB05290
      YD=YN                                                             SUB05300
      IF (XH.NE.X1) YD=Y1-(X1-XN)*(YH-Y1)/(XH-X1)                       SUB05310
      IF (YS.GE.YN) GO TO 2                                             SUB05320
      YD=YN                                                             SUB05330
      IF (YH.NE.Y1) XD=X1-(Y1-YN)*(XH-X1)/(YH-Y1)                       SUB05340
2     RETURN                                                            SUB05350
29    FORMAT (10F7.3)                                                   SUB05360
      END                                                               SUB05370
                                                                        SUB05380
C000012                                                                 SUB00010
C                                                                       SUB00020


      SUBROUTINE FREAD (NO,RO,NF,ZZ)                                    SUB00030
C                                                                       SUB00040
C    FREAD READS THE NUMERIC ARRAY WHICH IS USED FOR EQUATION 8 OF      SUB00050
C    SUBROUTINE FUNCT.  FIRST CARD IS NUMBER OF POINTS (N.GE.1),        SUB00060
C    FOLLOWED BY ONE POINT PER CARD.                                    SUB00070
C                                                                       SUB00080
      DIMENSION RO(101)                                                 SUB00090
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```
      READ (NF,89) NO,ZZ                                                SUB00100
      WRITE(6,90) NO,ZZ
      DO  10  I=1,NO
      READ(NF,88)  RO(I)
      WRITE(6,88)  RO(I)
 10   CONTINUE
89    FORMAT (I5,F9.3)                                                  SUB00120
 88   FORMAT(F8.5)
 90   FORMAT(1X,I5,F9.3)
      RETURN                                                            SUB00140
      END                                                               SUB00150
C000013                                                                 SUB00160
C                                                                       SUB00170


      SUBROUTINE GPRINT (G,NUMB)                                        SUB00180
C                                                                       SUB00190
C   GPRINT PRINTS THE DATA ARRAY 'G'  WHICH WAS INPUT TO                SUB00200
C   THE PROGRAM IN SUBROUTINE GARRAY.                                   SUB00210
C                                                                       SUB00220
      COMMON IMAX,JMAX,IIMX,JJMX,IJMX,ALPHA,SIZE,EPS,MODE,BOX,SD,IX,Z   SUB00230
      DIMENSION G(IJMX)                                                 SUB00240
      DIMENSION X(15)                                                   SUB00250
      DATA HYP,VERT/1H-,1H|/                                            SUB00260
      IF (NUMB.EQ.1) WRITE (6,99) MODE,Z                                SUB00270
      IF (NUMB.EQ.2) WRITE (6,92) Z                                     SUB00280
      JM2=JMAX/2                                                        SUB00290
      RIMAX=IMAX                                                        SUB00300
      RJMAX=JMAX                                                        SUB00310
      DX=SIZE/RIMAX                                                     SUB00320
      DXI=360./RJMAX                                                    SUB00330
C   INTRVL SETS THE NUMBER OF TERMS PRINTED PER LINE. IF IT IS ALTERED, SUB00340
C   ONE MUST ALSO REDIMENSION X AND ALTER FORMATS 98, 97, AND 95.       SUB00350
      INTRVL=15                                                         SUB00360
      IB=1                                                              SUB00370
      IT=IB+INTRVL-1                                                    SUB00380
1     IF (IT.GT.IMAX) IT=IMAX                                           SUB00390
      IBT=IT-IB+1                                                       SUB00400
      WRITE (6,98) (II,II=IB,IT)                                        SUB00410
      DO 2 I=1,IBT                                                      SUB00420
      RI=IB-1+I                                                         SUB00430
2     X(I)=-SIZE/2.+(RI-.5)*DX                                          SUB00440
      LM=7*IBT+1                                                        SUB00450
      WRITE (6,97) (X(I),I=1,IBT)                                       SUB00460
      WRITE (6,96) (HYP,L=1,LM),VERT                                    SUB00470
      JMH=JM2+1                                                         SUB00480
      DO 3 J=JMH,JMAX                                                   SUB00490
      RJ=J                                                              SUB00500
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```
      XI=-180.+DXI*(RJ-.5)                                                  SUB00510
      IGB=(J-1)*IMAX+IB                                                     SUB00520
      IGT=IGB-IB+IT                                                         SUB00530
3     WRITE (6,95) J,XI,(G(L),L=IGB,IGT)                                    SUB00540
      WRITE (6,94) (HYP,L=1,LM),VERT                                        SUB00550
      IB=IB+INTRVL                                                          SUB00560
      ITOLD=IT                                                              SUB00570
      IT=IT+INTRVL                                                          SUB00580
      IF (ITOLD.LT.IMAX) GO TO 1                                            SUB00590
      WRITE (6,93)                                                          SUB00600
99    FORMAT (1H1//'   THE ARRAY OF INPUT DATA (G), OBTAINED BY GARRAY,     SUB00610
     1MODE ',I1,', FOR Z='F7.3,' CM:')                                      SUB00620
98    FORMAT (///11X,'I= ',15I7)                                            SUB00630
97    FORMAT (11X,'X=  ',15F7.3)                                            SUB00640
96    FORMAT ('   J=     XI= |',115A1)                                      SUB00650
95    FORMAT (2X,I3,F9.2,'|',15F7.3)                                        SUB00660
94    FORMAT (14X,'|',115A1)                                                SUB00670
93    FORMAT (///5X)                                                        SUB00680
92    FORMAT (1H1//'   THE ADD-ON FUNCTION GARRAY FOR Z=',F7.3,' CM:')      SUB00690
      RETURN                                                                SUB00700
      END                                                                   SUB00710
C000014                                                                     SUB00720
C                                                                           SUB00730


      SUBROUTINE GPUNCH (Z,XO,YO,PHS,NOF,IMX,JMX,G)                         SUB00740
C                                                                           SUB00750
C   GPUNCH PUNCHES OUT THE FIRST NON-SYMMETRIC PORTION OF GARRAY            SUB00760
C   (OR WRITES IT ON FILE 7 IN CMS VERSION)                                 SUB00770
C                                                                           SUB00780
      COMMON /SYM/ ISM,JSM,MSM,FCU,IMS,JMS,QSM                              SUB00790
      DIMENSION G(IMX,JMX)                                                  SUB00800
      WRITE (7,39) NOF,IMX,JMX,ISM,JSM,IMS,JMS                              SUB00810
      WRITE (7,38) ((G(I,J),I=1,IMS),J=1,JMS)                               SUB00820
39    FORMAT(10I5)                                                          SUB00830
38    FORMAT(10F7.3)                                                        SUB00840
      RETURN                                                                SUB00850
      END                                                                   SUB00860
C000015                                                                     SUB00870
C                                                                           SUB00880


      SUBROUTINE READ (Z,XO,YO,PHISYM,NOF,IMAX,JMAX,G)                      SUB00890
C                                                                           SUB00900
C   READS THE NON-SYMMETRIC PORTION OF THE GARRAY AND EXPANDS IT TO AN      SUB00910
C   ORTHOGONAL SET. NOTE, INSURE SUFFICIENT DIMENSIONS IN MAIN PROGRAM.     SUB00920
C                                                                           SUB00930
      COMMON /SYM/ ISYM,JSYM,MSYM,FCU,IMS,JMS,QSYM                          SUB00940
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      COMMON /IO/ CMS,IN1,IN2,IN4                                          SUB00950
      DIMENSION G(IMAX,JMAX)                                               SUB00960
      READ (IN1,39) NOF,IMAX,JMAX,ISYM,JSYM,IMS,JMS                        SUB00970
      READ (IN1,38) Z,XO,YO,PHISYM                                         SUB00980
      READ (IN1,38) ((G(I,J),I=1,IMS),J=1,JMS)                             SUB00990
      WRITE(6,37) NOF,Z,XO,YO,PHISYM,IMAX,JMAX,JSYM                        SUB01000
      RJMX=JMAX                                                            SUB01010
      MSYM=JSYM                                                            SUB01020
      IF ((MSYM.EQ.0).OR.(MSYM.GT.JMAX)) MSYM=1                            SUB01030
      FCU=ISYM*JSYM*JMAX                                                   SUB01040
      IF (JSYM.GT.JMAX) FCU=JMAX                                           SUB01050
      QSYM=FCU/RJMX                                                        SUB01060
      DO 4 J=1,JMS                                                         SUB01070
      IF(ISYM.EQ.1) GO TO 2                                                SUB01080
      DO 1 I=1,IMS                                                         SUB01090
      II=IMAX+1-I                                                          SUB01100
1     G(II,J)=G(I,J)                                                       SUB01110
      GO TO 4                                                              SUB01120
2     J2=JMAX/2+1-J                                                        SUB01130
      J3=JMAX/2+J                                                          SUB01140
      J4=JMAX+1-J                                                          SUB01150
      DO 3 I=1,IMAX                                                        SUB01160
      II=IMAX+1-I                                                          SUB01170
      G(I,J2)=G(I,J)                                                       SUB01180
      G(II,J3)=G(I,J)                                                      SUB01190
3     G(II,J4)=G(I,J)                                                      SUB01200
4     CONTINUE                                                             SUB01210
39     FORMAT(10I5)                                                        SUB01220
38    FORMAT (10F7.3)                                                      SUB01230
37    FORMAT(///'  MODE 3 READS GARRAY DIRECTLY:   NOF='I4/'      Z='F7.3SUB01240
     1,',   XO=',F7.3,',   YO=',F7.3,',   PHISYM=',F7.3/'     IMAX=',I4,SUB01250
     2',   JMAX=',I4,',   JSYM=',I4///)                                   SUB01260
      RETURN                                                               SUB01270
      END                                                                  SUB01280
C000016                                                                    SUB01290
C                                                                          SUB01300


      SUBROUTINE MAP (IM,JM,A,N,Z,BAND)                                    SUB01310
C                                                                          SUB01320
C    MAP CALLS SUBROUTINE MIMPII AND PLOTS A CONTOUR MAP OF THE ARRAY      SUB01330
C                                                                          SUB01340
      DIMENSION A(IM,JM),T(24)                                             SUB01350
      DATA BL/1H /                                                         SUB01360
      DO 1 I=1,24                                                          SUB01370
1     T(I)=BL                                                              SUB01380
      ICON=1                                                               SUB01390
      IF(BAND.LT.0.) ICON=0                                                SUB01400
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      IF(BAND.LT.0.) BAND=-BAND                                          SUB01410
      AMIN=0.                                                            SUB01420
      IJT=0                                                              SUB01430
      AZ=1.                                                              SUB01440
      BZ=0.                                                              SUB01450
      WRITE(6,49) N,Z                                                    SUB01460
      CALL MTMPII (A,IM,JM,T,BAND,AZ,BZ,AMIN,IJT,ICON)                   SUB01470
49    FORMAT (1H1//'  THE FUNCTION SURFACE, TEST NO.'I3,'   Z='F5.3//)   SUB01480
      RETURN                                                             SUB01490
      END                                                                SUB01500
C000017                                                                  SUB01510
C                                                                        SUB01520


      SUBROUTINE GPLOT (G,GA,JMS)                                        SUB01530
C                                                                        SUB01540
C   GPLOT PRINTS A ROUGH PLOT OF THE LINE INTEGRAL FUNCTIONS IN GARRAY.  SUB01550
C                                                                        SUB01560
      COMMON IMAX,JMAX,IIMX,JJMX,IJMX,ALPHA,SIZE,EPS,MODE,BOX,SD,IX,Z    SUB01570
      COMMON /TAB/ INDEX(7),JSYM,ISYM                                    SUB01580
      COMMON /TAB2/ IPT,KPT,LPT,MPT,REST(5)                              SUB01590
      DIMENSION G(IMAX,JMAX),GA(IMAX,JMAX),ROW(101)                      SUB01600
      DIMENSION A(201),B(101),C(201),D(101)                              SUB01610
      JM=101                                                             SUB01620
      DATA BL,PL,ST,DH,EX/1H ,1H+,1H*,1H-,1HX/                           SUB01630
      JMS2=JMAX/2+1                                                      SUB01640
      IF(ISYM.EQ.2)JMS2=1                                                SUB01650
      JMS3=JMS2+JMS-1                                                    SUB01660
      DO 8 J=JMS2,JMS3                                                   SUB01670
      WRITE(6,67) (ST,I=1,120)                                           SUB01680
      DO 1 I=1,IMAX                                                      SUB01690
      A(I)=G(I,J)                                                        SUB01700
1     C(I)=GA(I,J)                                                       SUB01710
      AS=.5                                                              SUB01720
      BS=.0                                                              SUB01730
      CALL INTERP (A,IMAX,AS,B,JM,BS)                                    SUB01740
      CALL INTERP (C,IMAX,AS,D,JM,BS)                                    SUB01750
      WRITE (6,69) J                                                     SUB01760
      BIG=0.                                                             SUB01770
      SMALL=0.                                                           SUB01780
      DO 2 I=1,IMAX                                                      SUB01790
      IF(A(I).GT.BIG) BIG=A(I)                                           SUB01800
      IF(C(I).GT.BIG) BIG=C(I)                                           SUB01810
      IF(A(I).LT.SMALL) SMALL=A(I)                                       SUB01820
2     IF(C(I).LT.SMALL) SMALL=C(I)                                       SUB01830
      RANGE=BIG-SMALL                                                    SUB01840
      RINK=RANGE/80.                                                     SUB01850
      TOP=BIG+RINK                                                       SUB01860
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      CEN=BIG                                                          SUB01870
      BOT=BIG-RINK                                                     SUB01880
      KC=0                                                             SUB01890
      DO 7 K=1,41                                                      SUB01900
      IC=0                                                             SUB01910
      DO 6 I=1,101                                                     SUB01920
      ROW(I)=BL                                                        SUB01930
      IF((I.EQ.1).OR.(I.EQ.51).OR.(I.EQ.101))ROW(I)=PL                 SUB01940
      IF((K.EQ.1).OR.(K.EQ.41)) ROW(I)=PL                              SUB01950
      IF((TOP.GE.0.).AND.(BOT.LE.0.)) ROW(I)=DH                        SUB01960
      IF(IC.EQ.5) GO TO 3                                              SUB01970
      GO TO 4                                                          SUB01980
3     IC=0                                                             SUB01990
      IF(KC.EQ.10) ROW(I)=PL                                           SUB02000
4     IF(KPT.LE.2) GO TO 5                                             SUB02010
      IF((D(I).LE.TOP).AND.(D(I).GE.BOT)) ROW(I)=ST                    SUB02020
5     IF((B(I).LE.TOP).AND.(B(I).GE.BOT)) ROW(I)=EX                    SUB02030
6     IC=IC+1                                                          SUB02040
      IF(KC.EQ.5) KC=0                                                 SUB02050
      IF(KC.NE.0) WRITE (6,65) (ROW(I),I=1,101)                        SUB02060
      IF(KC.EQ.0) WRITE (6,68) CEN,(ROW(I),I=1,101)                    SUB02070
      TOP = TOP-2.*RINK                                                SUB02080
      CEN=CEN-2.*RINK                                                  SUB02090
      BOT=BOT-2.*RINK                                                  SUB02100
7     KC=KC+1                                                          SUB02110
8     WRITE (6,66) (ST,I=1,120)                                        SUB02120
69    FORMAT (//'      J=',I3///)                                      SUB02130
68    FORMAT (1X,F8.3,1X,101A1)                                        SUB02140
67    FORMAT (1H1,//121A1//)                                           SUB02150
66    FORMAT (///121A1///)                                             SUB02160
65    FORMAT (10X,101A1)                                               SUB02170
      RETURN                                                           SUB02180
      END                                                              SUB02190
C000018                                                                SUB02200
C                                                                      SUB02210


      SUBROUTINE INTERP (A,IM,AS,B,JM,BS)                              SUB02220
C                                                                      SUB02230
C   INTERP CONVERTS A REGULAR VECTOR A OF IM POINTS TO A REGULAR VECTOR SUB02240
C   B OF JM POINTS.  OS=.5 FOR A VECTOR WITH POINTS DEFINED IN THE     SUB02250
C   CENTER OF THE INTERVAL, AS AND BS ARE THE % OF AN INTERVAL FROM THE SUB02260
C   EDGE OF THE FIELD TO THE FIRST POINT (.0 OR .5 FOR EDGE OR CENTER  SUB02270
C   DEFINED POINTS)                                                    SUB02280
C                                                                      SUB02290
      DIMENSION A(IM),B(JM)                                            SUB02300
      RIM=IM                                                           SUB02310
      RJM=JM                                                           SUB02320
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      RAT=(RIM-1.+2.*AS)/(RJM-1.+2.*BS)                                 SUB02330
      DO 2 I=1,JM                                                       SUB02340
      BI=I                                                              SUB02350
      AI=RAT*(BI+BS)-AS                                                 SUB02360
      IA=AI                                                             SUB02370
      F=AI-FLOAT(IA)                                                    SUB02380
      IF((IA.EQ.0).OR.(IA.EQ.JM)) GO TO 1                               SUB02390
      B(I)=A(IA)+F*(A(IA+1)-A(IA))                                      SUB02400
      GO TO 2                                                           SUB02410
1     IF(IA.EQ.0) B(I)=A(1)*(F-AS)/(1.-AS)                              SUB02420
      IF(IA.EQ.JM) B(I)=A(JM)*F/(1.-AS)                                 SUB02430
2     CONTINUE                                                          SUB02440
      RETURN                                                            SUB02450
      END                                                               SUB02460
                                                                        SUB02470
C000019
C     ------------------------------------------------------------------SPL00010
C                                                                       SPL00020
C        SUBROUTINE SPLINE                                              SPL00030
C                                                                       SPL00040
C        PURPOSE                                                        SPL00050
C            PROVIDES INTERPOLATED VALUE USING "CUBIC SPLINE FITTING"   SPL00060
C                                                                       SPL00070
C        USAGE                                                          SPL00080
C            FIRST CALL TO SUBROUTINE:                                  SPL00090
C            CALL SPLINE(X,Y,M,XINT,YINT)                               SPL00100
C                                                                       SPL00110
C            SUBSEQUENT CALLS:                                          SPL00120
C            CALL SPLINN(X,Y,M,XINT,YINT)                               SPL00130
C                                                                       SPL00140
C        DESCRIPTION OF PARAMETERS                                      SPL00150
C            X:     MONOTONICALLY INCREASING ABSCISSA ARRAY             SPL00160
C            Y:     ONE-FOR-ONE CORRESPONDING ORDINATE ARRAY            SPL00170
C            M:     NUMBER OF X AND Y VALUES SUPPLIED < OR = 300        SPL00180
C            XINT:  VALUE OF ABSCISSA FOR WHICH CORRESPONDING ORDINATE  SPL00190
C                   IS TO BE INTERPOLATED (OR EXTRAPOLATED)             SPL00200
C            YINT:  INTERPOLATED (OR EXTRAPOLATED) ORDINATE VALUE       SPL00210
C                                                                       SPL00220
C                                                                       SPL00240
C        REMARKS                                                        SPL00250
C            IF SPECIFIED X FALLS OUTSIDE OF RANGE, AN EXTRAPOLATED     SPL00260
C            VALUE WILL BE SUPPLIED                                     SPL00270
C                                                                       SPL00280
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED                  SPL00290
C            SUBROUTINE SPLICO IS INCLUDED IN SUBROUTINE SPLIN PACKAGE  SPL00300
C                                                                       SPL00310
C        MATHEMATICAL METHOD                                            SPL00320
C            UPON FIRST ENTRY TO SPLIN, A CALL TO SPLICO IS MADE TO     SPL00330
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C            DETERMINE THE COEFFICIENTS TO BE USED IN PERFORMING THE    SPL00340
CC           INTERPOLATIONS. SEARCH FOR BRACKETING ABSCISSA VALUES IS   SPL00350
CCC          ALWAYS MADE FROM THE REFERENCE LAST USED IN INTERPOLATING. SPL00360
CCCC                                                                    SPL00370
CCCC       REFERENCE                                                    SPL00380
CCCC           PENNINGTON, RALPH H.,"INTRODUCTORY COMPUTER METHODS AND   SPL00390
CCCCC          NUMERICAL ANALYSIS", THE MACMILLAN COMPANY, NEW YORK, 1965 SPL00400
CCCCC                                                                   SPL00410
CCCCC    ---------------------------------------------------------------SPL00420
C        ---------------------------------------------------------------SPL00430

      SUBROUTINE SPLINE(X,Y,M,XINT,YINT)                                SPL00440
      DIMENSION X(M),Y(M),C(4,300)                                      SPL00460
      CALL SPLICO(X,Y,M,C)                                              SPL00470
      K=1                                                               SPL00480
      ENTRY SPLINN(X,Y,M,XINT,YINT)                                     SPL00490
    3 IF(XINT-X(1)) 70,1,2                                              SPL00500
   70 K=1                                                               SPL00510
      GO TO 7                                                           SPL00520
    1 YINT=Y(1)                                                         SPL00530
      RETURN                                                            SPL00540
    2 IF(XINT-X(K+1))6,4,5                                              SPL00550
    4 YINT=Y(K+1)                                                       SPL00560
      RETURN                                                            SPL00570
    5 K=K+1                                                             SPL00580
      IF(M-K) 71,71,3                                                   SPL00590
   71 K=M-1                                                             SPL00600
      GO TO 7                                                           SPL00610
    6 IF(XINT-X(K))13,12,11                                             SPL00620
   12 YINT=Y(K)                                                         SPL00630
      RETURN                                                            SPL00640
   13 K=K-1                                                             SPL00650
      GO TO 6                                                           SPL00660
    7 PRINT 101,XINT                                                    SPL00670
  101 FORMAT(8HOXINT = E18.9,32H, OUT OF RANGE FOR INTERPOLATION)       SPL00680
   11 YINT=(X(K+1)-XINT)*(C(1,K)*(X(K+1)-XINT)**2+C(3,K))               SPL00690
      YINT=YINT+(XINT-X(K))*(C(2,K)*(XINT-X(K))**2+C(4,K))              SPL00700
      RETURN                                                            SPL00710
      END                                                               SPL00720

      SUBROUTINE SPLICO(X,Y,M,C)                                        SPL00730
      DIMENSION X(M),Y(M),C(4,300),D(300),P(300),E(300),A(300,3),B(300),SPL00750
     1Z(300)                                                            SPL00760
      MM=M-1                                                            SPL00770
      DO 2 K=1,MM                                                       SPL00780
      D(K)=X(K+1)-X(K)                                                  SPL00790
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      P(K)=D(K)/6.                                                      SPL00800
    2 E(K)=(Y(K+1)-Y(K))/D(K)                                           SPL00810
      DO 3 K=2,MM                                                       SPL00820
    3 B(K)=E(K)-E(K-1)                                                  SPL00830
      A(1,2)=-1.-D(1)/D(2)                                              SPL00840
      A(1,3)=D(1)/D(2)                                                  SPL00850
      A(2,3)=P(2)-P(1)*A(1,3)                                           SPL00860
      A(2,2)=2.*(P(1)+P(2))-P(1)*A(1,2)                                 SPL00870
      A(2,3)=A(2,3)/A(2,2)                                              SPL00880
      B(2)=B(2)/A(2,2)                                                  SPL00890
      DO 4 K=3,MM                                                       SPL00900
      A(K,2)=2.*(P(K-1)+P(K))-P(K-1)*A(K-1,3)                           SPL00910
      B(K)=B(K)-P(K-1)*B(K-1)                                           SPL00920
      A(K,3)=P(K)/A(K,2)                                                SPL00930
    4 B(K)=B(K)/A(K,2)                                                  SPL00940
      Q=D(M-2)/D(M-1)                                                   SPL00950
      A(M,1)=1.+Q+A(M-2,3)                                              SPL00960
      A(M,2)=-Q-A(M,1)*A(M-1,3)                                         SPL00970
      B(M)=B(M-2)-A(M,1)*B(M-1)                                         SPL00980
      Z(M)=B(M)/A(M,2)                                                  SPL00990
      MN=M-2                                                            SPL01000
      DO 6 I=1,MN                                                       SPL01010
      K=M-I                                                             SPL01020
    6 Z(K)=B(K)-A(K,3)*Z(K+1)                                           SPL01030
      Z(1)=-A(1,2)*Z(2)-A(1,3)*Z(3)                                     SPL01040
      DO 7 K=1,MM                                                       SPL01050
      Q=1./(6.*D(K))                                                    SPL01060
      C(1,K)=Z(K)*Q                                                     SPL01070
      C(2,K)=Z(K+1)*Q                                                   SPL01080
      C(3,K)=Y(K)/D(K)-Z(K)*P(K)                                        SPL01090
    7 C(4,K)=Y(K+1)/D(K)-Z(K+1)*P(K)                                    SPL01100
      RETURN                                                            SPL01110
      END                                                               SPL01120

C000020
C                                                                       MET00010
C     ******************************************************************MET00020
C                                                                       MET00030
C     SUBROUTINE MTMPII                                                 MET00040
C                                                                       MET00050
C     PURPOSE                                                           MET00060
C               MTMPII WILL PRODUCE,ON THE PRINTER, A CONTOUR MAP       MET00070
C               OF ANY SINGLE PRECISION TWO DIMENSIONAL ARRAY.          MET00080
C                                                                       MET00090
C     USAGE                                                             MET00100
CC              CALL MTMPII(Y,N,M,T,BND,AZ,BZ,AMIN,IJT,ICON)            MET00110
C                                                                       MET00120
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C          DESCRIPTION OF PARAMETERS                                        METO0130
C                                                                           METO0140
C                    Y     - THE ARRAY TO BE CONTOURED. DIMENSIONED Y(N,M)METO0150
C                    N     - NUMBER OF ROWS IN Y.                         METO0160
C                    M     - NUMBER OF COLUMNS IN Y.                      METO0170
C                    T     - ARRAY FOR PLOT TITLE. REAL*4 T(24).          METO0180
C                    BND   - BANDWIDTH FOR THE CONTOURING. IF BND IS ZERO METO0190
C                            A BANDWIDTH WILL BE CALCULATED AS FOLLOWS    METO0200
C                            BND=(MAX(Y)-MIN(Y))/15.                      METO0210
C                    AZ    - A LINEAR TRANSFORMATION MAYBE PERFORMED ON   METO0220
C                            THE ARRAY Y OF THE FOLLOWING FORM AZ*Y+BZ.   METO0230
C                            IF AZ=0 THEN AZ WILL BE COMPUTED SUCH THAT   METO0240
C                            MAX(|MAX(Y)|,|MIM(Y)|) WILL BE LESS THAN 1,  METO0250
C                            AND BZ WILL BE LEFT AS INPUT.                METO0260
C                    BZ    - SEE UNDER AZ                                 METO0270
C                    AMIN  - THE LEVEL AT WHICH COUTOURING WILL BEGIN. IF METO0280
C                            AMIN > MIN(Y) THEN AMIN WILL BE CALCULATED   METO0290
C                            TO BE ,WITH REFERENCE AT ZERO, THE NEXT LOWERMETO0300
C                            CONTOUR LEVEL FROM MIN(Y) AS DETERMINED BY   METO0310
C                            BND.                                         METO0320
C                    IJT   - IF IJT=0 AMIN WILL BE CALCULATED AS DESCRIBEDMETO0330
C                            ABOVE.                                       METO0340
C                    ICON  - IF ICON=0 NO CONTOURING WILL BE DONE BUT THE METO0350
C                            ARRAY Y WILL BE PRINTED IN THE PLOT FORMAT.  METO0360
C                                                                         METO0370
C          REMARKS                                                        METO0380
C                    MTMPII REQUIRES A PRINTER WITH 132 PRINT POSITIONS.  METO0390
C                    IF NECESSARY THE MAP WILL BE SEGMENTED COLUMNWISE.   METO0400
C                    THE ROWS AND COLUMNS ARE NUMBERED ALONG THE EDGES SOMETO0410
C                    THAT A SEGMENTED MAP MAYBE EASILY JOINED TOGETHER.   METO0420
C                    ONLY THREE SIGNIFICANT FIGURES WILL BE PRINTED AT    METO0430
C                    EACH POINT. THE POSITION OF THE FIRST SIGNIFICANT    METO0440
C                    DIGIT WILL BE DETERMINED BY MAX(|MAX(Y)|,|MIN(Y)|).  METO0450
C                    THE PLOT WILL BE PRODUCED ON A INCH INCH GRID. IT    METO0460
C                    WILL BE ASSUMED THAT THE SPACING BETWEEN POINTS IN   METO0470
C                    BOTH DIRECTIONS IS THE SAME AND EQUAL FOR ALL POINTSMETO0480
C                                                                         METO0490
C          SUBROUTINES REQUIRED                                           METO0500
C                    NONE                                                 METO0510
C                                                                         METO0520
C          METHOD                                                         METO0530
C                    THE CONTOUR LEVELS ARE DETERMINED BY SIMPLE LINEAR   METO0540
C                    INTERPOLATION FROM THE FOUR SURROUNDING PIONTS.      METO0550
C                                                                         METO0560
C          ***************************************************************METO0570
C                                                                         METO0580
C                                                                         METO0590
C     MTMPII SUBROUTINE FOR ONE-INCH GRID SPACING                         METO0600
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C   OAKES CODE 5105   15 JAN 69                                          MET00610
C                                                                        MET00620

      SUBROUTINE MTMPII(Y,N,M,T,BND,AZ,BZ,AMIN,IJT,ICON)                 MET00630
      REAL*4 IH,KG,ITJZ                                                  MET00640
      DIMENSION A(140),B(140),C(140),D(140),IH(20),Y(N,M),TP(10),TPX(10)MET00650
     Z  ,TPM(10),XMT(10),BTM(10),BTX(10),BT(10),KG(10),T(24)             MET00660
      DIMENSION E(140),F(140),G(140),H(140)                              MET00670
C                                                                        MET00680
C                                                                        MET00690
      DATA DUE/4H    /,EPL/4H+   /,EMI/4H-   /,IH/1H0,1H ,1H1,1H ,1H2,  MET00700
     11H ,1H3,1H ,1H4,1H ,1H5,1H ,1H6,1H ,1H7,1H ,1H8,1H ,1H9,1H /,KG/   MET00710
     21H0,1H1,1H2,1H3,1H4,1H5,1H6,1H7,1H8,1H9/,BLK/4H    /              MET00720
C                                                                        MET00730
      YMIN=Y(1,1)                                                        MET00740
      YMAX=Y(1,1)                                                        MET00750
      DO 20 I=1,M                                                        MET00760
      DO 10 J=1,N                                                        MET00770
      YMIN=AMIN1(YMIN,Y(J,I))                                            MET00780
      YMAX=AMAX1(YMAX,Y(J,I))                                            MET00790
   10 CONTINUE                                                           MET00800
   20 CONTINUE                                                           MET00810
      DELY=YMAX-YMIN                                                     MET00820
      IF(BND) 25,25,30                                                   MET00830
   25 BND=DELY/15.0                                                      MET00840
   30 IF (AMIN-YMIN) 31,31,32                                            MET00850
   31 IF (IJT) 33,32,33                                                  MET00860
   32 PD=YMIN/BND                                                        MET00870
      PF=ABS(PD-INT(PD))                                                 MET00880
      IF (YMIN) 2,1,1                                                    MET00890
    1 AMIN=YMIN-PF*BND                                                   MET00900
      GO TO 33                                                           MET00910
    2 AMIN=YMIN-(1.0-PF)*BND                                             MET00920
   33 AHLD=AZ                                                            MET00930
      IF(AZ) 55,35,55                                                    MET00940
   35 SM=AMAX1(ABS(YMIN),ABS(YMAX))                                      MET00950
      NS=0                                                               MET00960
   40 NS=NS+1                                                            MET00970
      SM=10.0*SM                                                         MET00980
      IF(SM-1.0)40,50,45                                                 MET00990
   45 NS=NS-1                                                            MET01000
      SM=SM/10.0                                                         MET01010
      IF(SM-1.0)50,50,45                                                 MET01020
   50 AHLD=10.0**NS                                                      MET01030
   55 HBND=BND/2.0                                                       MET01040
      PRINT 70                                                           MET01050
      PRINT 6,T                                                          MET01060
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     6 FORMAT(5X,24A4,//)                                                    MET01070
       PRINT 57,AHLD,BZ                                                      MET01080
    57 FORMAT(1H0,65HTHE FOLLOWING TRANSFORMATION WAS PERFORMED ON THE INMET01090
      1PUT MATRIX    /5X,1H(,E12.5,8H*Y(I,J)+,E12.5,1H) //2X,73HAND THREEMET01100
      2 DIGITS TO THE RIGHT OF THE DECIMAL POINT ARE PRINTED IN THE MAP )MET01110
C                                                                          MET01120
       PRINT 54,YMAX,YMIN                                                  MET01130
    54 FORMAT(/4X,5HYMAX=,E15.7,5X,5HYMIN=,E15.7)                          MET01140
       IF (ICON)5,58,5                                                     MET01150
     5 PRINT 11,BND                                                        MET01160
    11 FORMAT(2X,17HTHE BAND WIDTH IS,E12.5,6H UNITS //4X,14HCONTOUR LEVEMET01170
      1LS       )                                                          MET01180
       I=0                                                                 MET01190
       YTOP=AMIN                                                           MET01200
       IF(ABS(YMIN-YMAX)-100.0*BND)53,53,58                                MET01210
    53 YB=YTOP                                                             MET01220
       YTOP=YTOP+BND                                                       MET01230
       I=I+1                                                               MET01240
       J=MOD(I,20)                                                         MET01250
       ITJZ=IH(J)                                                          MET01260
       IF(YB-YMAX)59,58,58                                                 MET01270
    59 PRINT 61,YB,YTOP,ITJZ                                               MET01280
    61 FORMAT(/4X,E10.3,4H TO ,E10.3,2H =,1X,A1)                           MET01290
       GO TO 53                                                            MET01300
    58 NCCP=0                                                              MET01310
       NCP=0                                                               MET01320
    60 PRINT 70                                                            MET01330
    70 FORMAT(1H1)                                                         MET01340
       PRINT 6,T                                                           MET01350
       NLINE=0                                                             MET01360
       NCCP=NCP+1                                                          MET01370
       NCP = NCP + 13                                                      MET01380
    73 IF(NCP-M)80,80,75                                                   MET01390
    75 NCP=M                                                               MET01400
    80 CONTINUE                                                            MET01410
       J=-2                                                                MET01420
       NLINE=NLINE+1                                                       MET01430
       LLINE=N-NLINE+1                                                     MET01440
C SET UP HEADING                                                           MET01450
       IF(NCCP-1) 85,85,90                                                 MET01460
    85 J = - 1                                                             MET01470
    90 DO 100 I = 1,135                                                    MET01480
       A(I)=BLK                                                            MET01490
       B(I)=BLK                                                            MET01500
       H(I) = BLK                                                          MET01510
   100 CONTINUE                                                            MET01520
   110 DO 160 L=NCCP,NCP                                                   MET01530
       J = J+8                                                             MET01540
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      KI=L                                                              MET01550
      IF(KI-100) 130,120,120                                            MET01560
  120 LL=KI/100                                                         MET01570
      A(J)=KG(LL+1)                                                     MET01580
      KI=KI-100*LL                                                      MET01590
      GO TO 135                                                         MET01600
  130 A(J)=KG(1)                                                        MET01610
  135 J=J+1                                                             MET01620
      IF(KI-10) 150,140,140                                             MET01630
  140 LL=KI/10                                                          MET01640
      A(J)=KG(LL+1)                                                     MET01650
      KI=KI-10*LL                                                       MET01660
      GO TO 155                                                         MET01670
  150 A(J)=KG(1)                                                        MET01680
  155 J=J+1                                                             MET01690
      A(J)=KG(KI+1)                                                     MET01700
  160 CONTINUE                                                          MET01710
C SETUP FIRST ROW OF ARRAY                                              MET01720
      GO TO 260                                                         MET01730
  170 NLINE=NLINE+1                                                     MET01740
      LLINE=N-NLINE+1                                                   MET01750
      IF(NLINE-N) 180,180,380                                           MET01760
  180 DO 190 I=1,135                                                    MET01770
      A(I)=BLK                                                          MET01780
      B(I)=BLK                                                          MET01790
      C(I)=BLK                                                          MET01800
      D(I)=BLK                                                          MET01810
      E(I) = BLK                                                        MET01820
      F(I) = BLK                                                        MET01830
      G(I) = BLK                                                        MET01840
      H(I) = BLK                                                        MET01850
  190 CONTINUE                                                          MET01860
      IF (ICON)195,260,195                                              MET01870
  195 NCY=NCCP-1                                                        MET01880
      J=4                                                               MET01890
      IF(NCY)200,200,210                                                MET01900
  200 J=5                                                               MET01910
  210 NCY=NCY+1                                                         MET01920
      IF(NCY-NCP) 220,220,260                                           MET01930
  220 IF(NCY-M) 230,260,260                                             MET01940
  230 NLINE = NLINE - 1                                                 MET01950
      YD1 = Y(NLINE,NCY) - Y(NLINE+1,NCY)                               MET01960
      YD2=Y(NLINE,NCY+1)-Y(NLINE+1,NCY+1)                               MET01970
      TP(1) = Y(NLINE,NCY)-0.125*YD1                                    MET01980
      TPX(1)=Y(NLINE,NCY)-0.250*YD1                                     MET01990
      TPM(1)=Y(NLINE,NCY)-0.375*YD1                                     MET02000
      XMT(1)=Y(NLINE,NCY)-0.500*YD1                                     MET02010
      BTM(1)=Y(NLINE,NCY)-0.625*YD1                                     MET02020
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      BTX(1)=Y(NLINE,NCY)-0.750*YD1                                     MET02030
      BT (1)=Y(NLINE,NCY)-0.875*YD1                                     MET02040
      TP (10)=Y(NLINE,NCY+1)-0.125*YD2                                  MET02050
      TPX(10)=Y(NLINE,NCY+1)-0.250*YD2                                  MET02060
      TPM(10)=Y(NLINE,NCY+1)-0.375*YD2                                  MET02070
      XMT(10)=Y(NLINE,NCY+1)-0.500*YD2                                  MET02080
      BTM(10)=Y(NLINE,NCY+1)-0.625*YD2                                  MET02090
      BTX(10)=Y(NLINE,NCY+1)-0.750*YD2                                  MET02100
      BT (10)=Y(NLINE,NCY+1)-0.875*YD2                                  MET02110
      NLINE = NLINE + 1                                                 MET02120
      D1=0.1 * (TP (10)-TP (1))                                         MET02130
      D2=0.1 * (TPX(10)-TPX(1))                                         MET02140
      D3=0.1 * (TPM(10)-TPM(1))                                         MET02150
      D4=0.1 * (XMT(10)-XMT(1))                                         MET02160
      D5=0.1 * (BTM(10)-BTM(1))                                         MET02170
      D6=0.1 * (BTX(10)-BTX(1))                                         MET02180
      D7=0.1 * (BT (10)-BT (1))                                         MET02190
      DO 240 I = 2,9                                                    MET02200
      TP (I)=TP (I-1)+D1                                                MET02210
      TPX(I)=TPX(I-1)+D2                                                MET02220
      TPM(I)=TPM(I-1)+D3                                                MET02230
      XMT(I)=XMT(I-1)+D4                                                MET02240
      BTM(I)=BTM(I-1)+D5                                                MET02250
      BTX(I)=BTX(I-1)+D6                                                MET02260
      BT (I)=BT (I-1)+D7                                                MET02270
  240 CONTINUE                                                          MET02280
      DO 250 I=1,10                                                     MET02290
      J=J+1                                                             MET02300
      I1=MOD(IFIX((TP (I)-AMIN)/BND),20)+1                              MET02310
      I2=MOD(IFIX((TPX(I)-AMIN)/BND),20)+1                              MET02320
      I3=MOD(IFIX((TPM(I)-AMIN)/BND),20)+1                              MET02330
      I4=MOD(IFIX((XMT(I)-AMIN)/BND),20)+1                              MET02340
      I5=MOD(IFIX((BTM(I)-AMIN)/BND),20)+1                              MET02350
      I6=MOD(IFIX((BTX(I)-AMIN)/BND),20)+1                              MET02360
      I7=MOD(IFIX((BT (I)-AMIN)/BND),20)+1                              MET02370
      A(J)=IH(I1)                                                       MET02380
      B(J)=IH(I2)                                                       MET02390
      C(J)=IH(I3)                                                       MET02400
      D(J)=IH(I4)                                                       MET02410
      E(J)=IH(I5)                                                       MET02420
      F(J)=IH(I6)                                                       MET02430
      G(J)=IH(I7)                                                       MET02440
  250 CONTINUE                                                          MET02450
      GO TO 210                                                         MET02460
  260 NCY=NCCP-1                                                        MET02470
      J=-2                                                              MET02480
      IF(NCY) 265,265,270                                               MET02490
  265 J=-1                                                              MET02500
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      GO TO 330                                                          MET02510
  270 NCY=NCY+1                                                          MET02520
      IF(NCY-NCP) 280,280,310                                            MET02530
  280 J=J+7                                                              MET02540
      THLD=AHLD*Y(NLINE,NCY)+BZ                                          MET02550
      IF(THLD) 285,290,290                                               MET02560
  285 H(J)=EMI                                                           MET02570
      GO TO 295                                                          MET02580
  290 H(J)=EPL                                                           MET02590
  295 NUM=INT(ABS(THLD-INT(THLD))*1000.0+0.5)                            MET02600
      NDS=100                                                            MET02610
      DO 300 KK=1,3                                                      MET02620
      J=J+1                                                              MET02630
      KI=NUM/NDS                                                         MET02640
      H(J)=KG(KI+1)                                                      MET02650
      NUM=NUM-KI*NDS                                                     MET02660
      NDS=NDS/10                                                         MET02670
  300 CONTINUE                                                           MET02680
      GO TO 270                                                          MET02690
  310 IF(NCP-M) 360,320,320                                              MET02700
  320 IF(J-127)330,330,360                                               MET02710
  330 J=J+3                                                              MET02720
      KI=NLINE                                                           MET02730
      IF(KI-100) 340,335,335                                             MET02740
  335 LL=KI/100                                                          MET02750
      H(J)=KG(LL+1)                                                      MET02760
      KI=KI-100*LL                                                       MET02770
      GO TO 343                                                          MET02780
  340 H(J)=KG(1)                                                         MET02790
  343 J=J+1                                                              MET02800
      IF(KI-10) 350,345,345                                              MET02810
  345 LL=KI/10                                                           MET02820
      H(J)=KG(LL+1)                                                      MET02830
      KI=KI-10*LL                                                        MET02840
      GO TO 355                                                          MET02850
  350 H(J)=KG(1)                                                         MET02860
  355 J=J+1                                                              MET02870
      H(J)=KG(KI+1)                                                      MET02880
      J=J-5                                                              MET02890
      IF(NCY-1) 270,270,360                                              MET02900
  360 IF(NLINE-1)362,362,368                                             MET02910
  362 PRINT 370,(A(I),I=1,132),(B(IP1),IP1=1,132),(H(IP2),IP2=1,132)     MET02920
      GO TO 170                                                          MET02930
  368 PRINT 370,(A(I),I=1,132),(B(IP1),IP1=1,132),(C(IP2),IP2=1,132),    MET02940
     1(D(IP3),IP3=1,132),(E(IP4),IP4=1,132),(F(IP5),IP5=1,132),          MET02950
     2(G(IP6),IP6=1,132),(H(IP7),IP7=1,132)                              MET02960
  370 FORMAT(132A1)                                                      MET02970
      GO TO 170                                                          MET02980
```

```
  380 DO 390 I=1,135                                                    MET02990
      A(I)=BLK                                                          MET03000
      B(I)=BLK                                                          MET03010
      C(I)=BLK                                                          MET03020
      D(I)=BLK                                                          MET03030
  390 CONTINUE                                                          MET03040
      J=-2                                                              MET03050
      IF(NCCP-1) 395,395,400                                            MET03060
  395 J=-1                                                              MET03070
  400 DO 430 L=NCCP,NCP                                                 MET03080
      J=J+8                                                             MET03090
      KI=L                                                              MET03100
      IF(KI-100) 410,405,405                                            MET03110
  405 LL=KI/100                                                         MET03120
      C(J)=KG(LL+1)                                                     MET03130
      KI=KI-100*LL                                                      MET03140
      GO TO 412                                                         MET03150
  410 C(J)=KG(1)                                                        MET03160
  412 J=J+1                                                             MET03170
      IF(KI-10) 420,415,415                                             MET03180
  415 LL=KI/10                                                          MET03200
      C(J)=KG(LL+1)                                                     MET03210
      KI=KI-10*LL                                                       MET03220
      GO TO 422                                                         MET03230
  420 C(J)=KG(1)                                                        MET03240
  422 J=J+1                                                             MET03250
      C(J)=KG(KI+1)                                                     MET03260
  430 CONTINUE                                                          MET03270
      PRINT 370, (B(IP1),IP1=1,132),(C(IP2),IP2=1,132)                  MET03280
      IF(NCP-M)60,500,500                                               MET03290
  500 RETURN                                                            MET03300
      END

C000021
C                                                                       GAUS  10
C     .................................................................GAUS  20
C                                                                       GAUS  30
C        SUBROUTINE GAUSS                                               GAUS  40
C                                                                       GAUS  50
C        PURPOSE                                                        GAUS  60
C           COMPUTES A NORMALLY DISTRIBUTED RANDOM NUMBER WITH A GIVEN  GAUS  70
C           MEAN AND STANDARD DEVIATION                                 GAUS  80
C                                                                       GAUS  90
C        USAGE                                                          GAUS 100
C           CALL GAUSS(IX,S,AM,V)                                       GAUS 110
C                                                                       GAUS 120
C        DESCRIPTION OF PARAMETERS                                      GAUS 130
```

```
C            IX -IX MUST CONTAIN AN ODD INTEGER NUMBER WITH NINE OR      GAUS 140
C                LESS DIGITS ON THE FIRST ENTRY TO GAUSS. THEREAFTER     GAUS 150
C                IT WILL CONTAIN A UNIFORMLY DISTRIBUTED INTEGER RANDOM  GAUS 160
C                NUMBER GENERATED BY THE SUBROUTINE FOR USE ON THE NEXT  GAUS 170
C                ENTRY TO THE SUBROUTINE.                                GAUS 180
C            S  -THE DESIRED STANDARD DEVIATION OF THE NORMAL            GAUS 190
C                DISTRIBUTION.                                           GAUS 200
C            AM -THE DESIRED MEAN OF THE NORMAL DISTRIBUTION             GAUS 210
C            V  -THE VALUE OF THE COMPUTED NORMAL RANDOM VARIABLE        GAUS 220
C                                                                        GAUS 230
C         REMARKS                                                        GAUS 240
C            THIS SUBROUTINE USES RANDU WHICH IS MACHINE SPECIFIC        GAUS 250
C                                                                        GAUS 260
C         SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED                  GAUS 270
C            RANDU                                                       GAUS 280
C                                                                        GAUS 290
C         METHOD                                                         GAUS 300
C            USES 12 UNIFORM RANDOM NUMBERS TO COMPUTE NORMAL RANDOM     GAUS 310
C            NUMBERS BY CENTRAL LIMIT THEOREM. THE RESULT IS THEN        GAUS 320
C            ADJUSTED TO MATCH THE GIVEN MEAN AND STANDARD DEVIATION.    GAUS 330
C            THE UNIFORM RANDOM NUMBERS COMPUTED WITHIN THE SUBROUTINE   GAUS 340
C            ARE FOUND BY THE POWER RESIDUE METHOD.                      GAUS 350
C                                                                        GAUS 360
C     ..................................................................GAUS 370
C                                                                        GAUS 380


      SUBROUTINE GAUSS(IX,S,AM,V)                                        GAUS 390
      A=0.0                                                              GAUS 400
      DO 50 I=1,12                                                       GAUS 410
      CALL RANDU(IX,IY,Y)                                                GAUS 420
      IX=IY                                                              GAUS 430
   50 A=A+Y                                                              GAUS 440
      V=(A-6.0)*S+AM                                                     GAUS 450
      RETURN                                                             GAUS 460
      END                                                                GAUS 470


C000022
C                                                                        RAND  10
C     ..................................................................RAND  20
C                                                                        RAND  30
C        SUBROUTINE RANDU                                                RAND  40
C                                                                        RAND  50
C        PURPOSE                                                         RAND  60
C           COMPUTES UNIFORMLY DISTRIBUTED RANDOM REAL NUMBERS BETWEEN   RAND  70
C           0 AND 1.0 AND RANDOM INTEGERS BETWEEN ZERO AND               RAND  80
C           2**31. EACH ENTRY USES AS INPUT AN INTEGER RANDOM NUMBER     RAND  90
```

```
C           AND PRODUCES A NEW INTEGER AND REAL RANDOM NUMBER.                RAND 100
C                                                                             RAND 110
C       USAGE                                                                 RAND 120
C          CALL RANDU(IX,IY,YFL)                                              RAND 130
C                                                                             RAND 140
C       DESCRIPTION OF PARAMETERS                                             RAND 150
C          IX - FOR THE FIRST ENTRY THIS MUST CONTAIN ANY ODD INTEGER         RAND 160
C               NUMBER WITH NINE OR LESS DIGITS. AFTER THE FIRST ENTRY,       RAND 170
C               IX SHOULD BE THE PREVIOUS VALUE OF IY COMPUTED BY THIS        RAND 180
C               SUBROUTINE.                                                   RAND 190
C          IY - A RESULTANT INTEGER RANDOM NUMBER REQUIRED FOR THE NEXT       RAND 200
C               ENTRY TO THIS SUBROUTINE. THE RANGE OF THIS NUMBER IS         RAND 210
C               BETWEEN ZERO AND 2**31                                        RAND 220
C          YFL- THE RESULTANT UNIFORMLY DISTRIBUTED, FLOATING POINT,          RAND 230
C               RANDOM NUMBER IN THE RANGE 0 TO 1.0                           RAND 240
C                                                                             RAND 250
C       REMARKS                                                               RAND 260
C          THIS SUBROUTINE IS SPECIFIC TO SYSTEM/360 AND WILL PRODUCE         RAND 270
C          2**29 TERMS BEFORE REPEATING.  THE REFERENCE BELOW DISCUSSESRAND 280
C          SEEDS (65539 HERE), RUN PROBLEMS, AND PROBLEMS CONCERNING          RAND 290
C          RANDOM DIGITS USING THIS GENERATION SCHEME.  MACLAREN AND           RAND 300
C          MARSAGLIA, JACM 12, P. 83-89, DISCUSS CONGRUENTIAL                 RAND 310
C          GENERATION METHODS AND TESTS.  THE USE OF TWO GENERATORS OF        RAND 320
C          THE RANDU TYPE, ONE FILLING A TABLE AND ONE PICKING FROM THERAND 330
C          TABLE, IS OF BENEFIT IN SOME CASES.  65549 HAS BEEN                RAND 340
C          SUGGESTED AS A SEED WHICH HAS BETTER STATISTICAL PROPERTIES RAND 350
C          FOR HIGH ORDER BITS OF THE GENERATED DEVIATE.                      RAND 360
C          SEEDS SHOULD BE CHOSEN IN ACCORDANCE WITH THE DISCUSSION           RAND 370
C          GIVEN IN THE REFERENCE BELOW.  ALSO, IT SHOULD BE NOTED THATRAND 380
C          IF FLOATING POINT RANDOM NUMBERS ARE DESIRED,AS ARE                RAND 390
C          AVAILABLE FROM RANDU, THE RANDOM CHARACTERISTICS OF THE            RAND 400
C          FLOATING POINT DEVIATES ARE MODIFIED AND IN FACT THESE             RAND 410
C          DEVIATES HAVE HIGH PROBABILITY OF HAVING A TRAILING LOW            RAND 420
C          ORDER ZERO BIT IN THEIR FRACTIONAL PART.                           RAND 430
C                                                                             RAND 440
C       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED                         RAND 450
C          NONE                                                               RAND 460
C                                                                             RAND 470
C       METHOD                                                                RAND 480
C          POWER RESIDUE METHOD DISCUSSED IN IBM MANUAL C20-8011,             RAND 490
C          RANDOM NUMBER GENERATION AND TESTING                               RAND 500
C                                                                             RAND 510
C     ..........................................................................RAND 520
C                                                                             RAND 530
```

```
      SUBROUTINE RANDU(IX,IY,YFL)                                    RAND 540
      IY=IX*65539                                                    RAND 550
      IF(IY)5,6,6                                                    RAND 560
    5 IY=IY+2147483647+1                                             RAND 570
    6 YFL=IY                                                         RAND 580
      YFL=YFL*.4656613E-9                                            RAND 590
      RETURN                                                         RAND 600
      END
```

```
//  EXEC FORTCLGP,TIME.GO=3,REGION.GO=100K
//FORT.SYSIN  DD *
C*****************************************************************
C     COMPUTER  PROGRAM  1
C*****************************************************************
C
C
C
C     THIS COMPUTER PROGRAM IS DESIGNED TO INTERPOLATE THE DENSITY
C     VALUES AS OBTAINED FROM THE AGARD 167 TABLES AND INPUT ALONG
C     A LINE OF SIGHT THAT PASSES THROUGH THE CONE AXIS. SUBROUTINE
C     LSQPL2 FROM THE IBM SUBROUTINE PACKAGE IS USED TO PASS A TENTH
C     DEGREE POLYNOMIAL THROUGH THE INPUT VALUES AND THIS IS THEN
C     USED TO OBTAIN THE DENSITY AT 201 EQUIDISTANT POINTS ALONG THE
C     LINE OF SIGHT. THESE 201 VALUES ARE PUNCHED OUT ACCORDING TO
C     THE FORMAT REQUIRED FOR INPUT INTO SUBROUTINE FREAD OF COMPUTER
C     PROGRAM HOLOVERT.  ANY NUMBER OF OF LINES OF SIGHT MAY BE READ
C     IN AT A TIME BY SPECIFYING THE VALUE OF 'W' AS 1.0 ACCORDING
C     TO FORMAT 60 IN THE DATA CARDS READ IN.THE DATA CARDS TO BE
C     READ IN ARE AS FOLLOWS
C
C     CARD 1... UPPER AND LOWER SHOCK WAVE ANGLES IN THE ORDER IN
C               WHICH THE DENSITY IS READ IN .... FORMAT 10
C
C     CARDS 2 TO 5 .... DENSITY VALUES ALONG A LINE OF SIGHT AS
C                       OBTAINED FROM AGARD 167 .....FORMAT 30
C     CARD 6 ..... THE VALUE OF 'W'. IF MORE THAN ONE LINE OF SIGHT
C                  IS BEING INPUT, SET 'W' EQUAL TO 1.0. IF NO DATA SET
C                  FOLLOWS, SET 'W' EQUAL TO ANY OTHER VALUE...FORMAT 60
C
C     NOTE... THE CARDS PUNCHED OUT FOR THE VARIOUS LINES OF SIGHT
C             ARE SEPARATED BY A CARD HAVING FORMAT 43. THIS CARD
C             MUST BE REMOVED BEFORE THE DATA IS FED IN INTO
C             SUBROUTINE FREAD.
C
C
C
//GO.FT06F001  DD  DCB=BLKSIZE=133
//GO.SYSIN  DD *
      DIMENSION AX(100),X(28),RHO(28),WI(28),RHOP(28),DELY(28),B(30),
     1SB(30),XP(201),ARHO(201)
      REAL*8 TITLE(10)/10*' '/
      REAL*8 B,X,RHO,WI,RHOP,DELY,SB,ARHO,Y,XP,T
      REAL*8 A1,A2,A3,A4,A5,A6,A7,A8,A9,A10,A11
      Y(A1,A2,A3,A4,A5,A6,A7,A8,A9,A10,A11,T)=A1+A2*T+A3*T**2+A4*T**3+
     1A5*T**4+A6*T**5+A7*T**6+A8*T**7+A9*T**8+A10*T**9+A11*T**10
      PIE=3.14159265
      R=1.0
```

```
 1  READ(5,10) U,V
    SU=(U*PIE)/180.
    Z=.95/TAN(SU)
    SL=(V*PIE)/180.
    THETA=(29.677*PIE)/180.
    R2=Z*TAN((10.*PIE)/180.)
    R3=Z*TAN(SL)
    R1=Z*TAN(SU)
    R4=(1.0-R3)
    R5=(1.0-R2)
    R6=(1.0+R2)
    R7=(1.0+R1)
    DELR1=(R1-R2)/10.
    DELR2=(R3-R2)/10.
    DELR3=(1.0-R1)/25.
    DELR4=(1.0-R3)/25.
    DELR5=R2/12.5
    RHO(1)=0.
    READ(5,30) (RHO(I),I=2,14)
    READ(5,30) (RHO(I),I=15,27)
    RHO(28)=0.
    AX(1)=0.
    DO 2 I=2,26
 2  AX(I)= AX(I-1)+DELR4
    X(1)=0.
    X(2)=AX(26)
    DO 3 I=27,35
    AX(I)=AX(I-1)+DELR2
    J=I-24
 3  X(J)=AX(I)
    AX(36)=AX(35)+DELR2/2.
    X(12)=AX(36)
    DO 31 I=37,38
    AX(I)=AX(I-1)+DELR2/4.
    J=I-24
31  X(J)=AX(I)
    DO 4 I=39,63
 4  AX(I)=AX(I-1)+DELR5
    X(15)=AX(63)
    DO 5 I=64,65
    AX(I)=AX(I-1)+DELR1/4.
    J=I-48
 5  X(J)=AX(I)
    AX(66)=AX(65)+DELR1/2.
    X(18)=AX(66)
    DO 32 I=67,75
    AX(I)=AX(I-1)+DELR1
    J=I-48
```

```
 32   X(J)=AX(I)
      X(28)=2.0
      DO 6 I=1,28
  6   WI(I)=1.0
      CALL LSQPL2(28,-10,X,RHO,WI,RHOP,DELY,B,SB,TITLE)
      WRITE(6,50)
 50   FORMAT(////'  LSQPL2  RETURNS')
 10   FORMAT(2F10.7)
 30   FORMAT(10F8.5,/,3F8.5)
      A1=B(1)
      A2=B(2)
      A3=B(3)
      A4=B(4)
      A5=B(5)
      A6=B(6)
      A7=B(7)
      A8=B(8)
      A9=B(9)
      A10=B(10)
      A11=B(11)
      ARHO(1)=0.
      DELX=2./200.
      XP(1)=0.
      DO  7 I=2,201
      XP(I)=XP(I-1)+DELX
         IF(XP(I).LT.R4) GO TO 9
      IF(XP(I).GT.R7) GO TO 9
 51   FORMAT(///'  COMPUTING ARHO(I)')
      ARHO(I)=Y(A1,A2,A3,A4,A5,A6,A7,A8,A9,A10,A11,XP(I))
      GO TO 7
  9   ARHO(I)=0.
  7   CONTINUE
      WRITE(6,41)
      WRITE(6,40) (I,X(I),RHO(I),I=1,28)
      WRITE(6,49)
      WRITE(6,40) (I,XP(I),ARHO(I),I=1,201)
      WRITE(7,42) (ARHO(I),I=1,201)
      WRITE(7,43)
      READ(5,60) W
      IF(W.EQ.1.) GO TO 1
 43   FORMAT('XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX')
 60   FORMAT(F5.0)
 39   FORMAT(/////)
 42   FORMAT(F8.5)
 49   FORMAT(////,5X,'I',10X,'XP(I)',10X,'ARHO(I)',///)
 40   FORMAT(5X,I3,10X,F10.5,10X,F10.5)
 41   FORMAT(////,5X,'I',10X,'X(I)',10X,'RHO(I)',///)
      STOP
```

```
      END
C
C
C     A SAMPLE DATA DECK
C
C
22.696     22.696
.1555    .1913    .2168    .2383    .2574    .2749    .2908    .3053    .3178    .3271
.3300    .3308    .3310
.3310    .3308    .3300    .3271    .3178    .3053    .2908    .2749    .2574    .2383
.2168    .1913    .1555
1.
22.99      22.99
.1482    .1837    .2089    .2300    .2489    .2661    .2802    .2964    .3088    .3181
.3210    .3219    .3221
.3221    .3219    .3210    .3181    .3088    .2964    .2802    .2661    .2489    .2300
.2089    .1837    .1482
5.
```

```
C*****************************************************************
C      COMPUTER  PROGRAM  2
C*****************************************************************
C
C
C
C      THIS COMPUTER PROGRAM IS DESIGNED TO INTERPOLATE THE DENSITY
C      VALUES GIVEN IN AGARD 167 FOR MACH NUMBERS BETWEEN 2.5 AND 3.0.
C      INTERPOLATION IS DONE BY CALLING ON SUBROUTINE LSQPL2 OF THE
C      IBM SUBROUTINE PACKAGE TO PASS A CUBIC CURVE THROUGH THE FOUR
C      INPUT VALUES OF DENSITY AT MACH NUMBERS 1.5,2.0,3.0 AND 4.0.
C      THE VALUES OF THE DENSITY FOR MACH NUMBERS FROM 2.5 TO 3.0
C      IN INCREMENTS OF M=.01 ARE THEN EVALUATED AND PRINTED FOR
C      EACH SET OF INPUT DATA VALUES.
C
C      INPUT DATA...
C
C      EACH CARD HAS 4 DENSITY VALUES CORRESPONDING TO THE VALUES
C      AT M=1.5,2.0,3.0,AND 4.0 IN THAT ORDER  ACCORDING TO FORMAT 10.
C      FOR EACH VALUE OF 'PHI' THERE ARE 14 SUCH CARDS, THE LAST ONE
C      BEING THAT OF THE SHOCK ANGLE. CARDS ARE INPUT IN THE ORDER
C      OF INCREASING 'XI' FOR EACH 'PHI' AND IN THE ORDER OF INCREASING
C      'PHI'.
C
C
//   EXEC FORTCLGP,TIME.GO=5,REGION.GO=100K
//FORT.SYSIN  DD *
      DIMENSION AM(4),RO(4),WI(10),AROP(10),DELY(10),B(10),SB(10),
     1AAM(50),ROP(50),PHI(10)
      REAL*8  TITLE(10)/10*' '/
      REAL*8  B,AM,RO,WI,AROP,DELY,SB,AAM,ROP,T,A1,A2,A3,A4
      FUN(A1,A2,A3,A4,T)=A1+A2*T+A3*T**2+A4*T**3
      CALL ERRSET(209,256,-1,1,0,203)
      PHI(1)=0.
      DELPHI=22.5
      DO 2 K=1,10
      WRITE(6,30) PHI(K)
   30 FORMAT(////,1X,'PHI=',F10.5,///)
      PHI(K+1)=PHI(K)+DELPHI
      AXI=0.
      DELXI=0.1
      AM(1)=1.5
      AM(2)=2.0
      AM(3)=3.0
      AM(4)=4.
      DO 2 I=1,14
      READ(5,10) (RO(M),M=1,4)
   10 FORMAT(4F10.5)
```

```
      WRITE(6,40) AXI
   40 FORMAT(///,1X,'XI=',F10.5,///)
      AXI=AXI+DELXI
      DO 3 L=1,10
    3 WI(L)=1.0D0
      WRITE(6,10) (RO(II),II=1,4)
      WRITE(6,10) (AM(II),II=1,4)
      WRITE(6,10) (WI(II),II=1,10)
      CALL LSQPL2(4,-3,AM,RO,WI,AROP,DELY,B,SB,TITLE)
      A1=B(1)
      A2=B(2)
      A3=B(3)
      A4=B(4)
      WRITE(6,50)
   50 FORMAT(///,3X,'MACH NO.',5X,'DENSITY',///)
      AAM(1)=2.5
      DLM=.01
      DO 2 J=1,50
      ROP(J)=FUN(A1,A2,A3,A4,AAM(J))
      WRITE(6,20) AAM(J),ROP(J)
   20 FORMAT(1X,F15.7,10X,F15.7)
      AAM(J+1)=AAM(J)+DLM
    2 CONTINUE
      STOP
      END
C
C     A TYPICAL DATA DECK ......
C
//GO.FT06F001 DD SPACE=(CYL,(10,2))
//GO.SYSIN  DD *
1.6252     1.9494     2.6869     3.3792
1.6244     1.9490     2.6866     3.3790
1.6223     1.9478     2.6859     3.3784
1.6154     1.9437     2.6830     3.3760
1.5954     1.9296     2.6726     3.3669
1.5717     1.9102     2.6567     3.3524
1.5468     1.8873     2.6363     3.3332
1.5215     1.8618     2.6118     3.3094
1.4961     1.8342     2.5836     3.2813
1.4705     1.8047     2.5519     3.2491
1.4444     1.7730     2.5166     3.2126
1.4172     1.7387     2.4774     3.1715
1.3877     1.7008     2.4338     3.1256
2.2427     1.7343     1.4003     1.2829
1.6004     1.9109     2.6169     3.2783
1.6006     1.9115     2.6197     3.2857
1.5993     1.9111     2.6203     3.2870
1.5938     1.9082     2.6195     3.2887
```

1.5761	1.8964	2.6125	3.2850
1.5541	1.8790	2.5996	3.2752
1.5306	1.8579	2.5819	3.2601
1.5065	1.8340	2.5601	3.2404
1.4822	1.8079	2.5345	3.2163
1.4576	1.7798	2.5053	3.1879
1.4324	1.7495	2.4725	3.1553
1.4060	1.7167	2.4358	3.1182
1.3773	1.6802	2.3948	3.0763
2.2591	1.7461	1.4071	1.2873
1.5329	1.8049	2.4243	3.0007
1.5355	1.8083	2.4354	3.0284
1.5364	1.8098	2.4394	3.0365
1.5347	1.8105	2.4441	3.0470
1.5228	1.8048	2.4459	3.0572
1.5054	1.7926	2.4407	3.0591
1.4857	1.7762	2.4301	3.0549
1.4648	1.7566	2.4151	3.0454
1.4434	1.7345	2.3961	3.0314
1.4213	1.7102	2.2734	3.013
1.3985	1.6836	2.3472	2.9904
1.3744	1.6544	2.3172	2.9635
1.3477	1.6215	2.2829	2.9319
2.3073	1.7811	1.4278	1.3009
1.4409	1.6579	2.1550	2.6124
1.4467	1.6648	2.1771	2.6669
1.4503	1.6690	2.1856	2.6833
1.4533	1.6742	2.1971	2.7049
1.4488	1.6762	2.2097	2.7319
1.4372	1.6706	2.2137	2.7474
1.4222	1.6600	2.2116	2.7556
1.4055	1.6458	2.2047	2.7582
1.3877	1.6288	2.1936	2.7553
1.3690	1.6093	2.1788	2.7490
1.3492	1.5874	2.1604	2.7382
1.3279	1.5627	2.1384	2.7232
1.3040	1.5343	2.1124	2.7041
2.3838	1.8380	1.4620	1.3235
1.3491	1.5057	1.8691	2.1983
1.3574	1.5160	1.9030	2.2807
1.3632	1.5224	1.9155	2.3043
1.3699	1.5313	1.9325	2.3352
1.3713	1.5394	1.9533	2.3745
1.3644	1.5391	1.9641	2.3997
1.3536	1.5333	1.9683	2.4165
1.3406	1.5237	1.9394	2.4271
1.3260	1.5110	1.9623	2.4323
1.3104	1.4957	1.9534	2.4342

1.2934	1.4778	1.9412	2.4318
1.2747	1.4570	1.9255	2.4257
1.2529	1.4323	1.9060	2.4159
2.4815	1.9132	1.5083	1.3543
1.2803	1.3838	1.6259	1.8384
1.2891	1.3962	1.6709	1.9486
1.2953	1.4033	1.6854	1.9764
1.3026	1.4131	1.7044	2.0111
1.3057	1.4227	1.7269	2.0531
1.3010	1.4240	1.7387	2.0788
1.2924	1.4201	1.7440	2.0958
1.2816	1.4125	1.7444	2.1069
1.2692	1.4020	1.7409	2.1133
1.2555	1.3889	1.7339	2.1158
1.2404	1.3732	1.7237	2.1149
1.2233	1.3544	1.7102	2.1109
1.2027	1.3310	1.6930	2.1036
2.5885	1.9992	1.5626	1.3902
1.2439	1.3123	1.4669	1.5914
1.2509	1.3255	1.5244	1.7339
1.2551	1.3312	1.5388	1.7631
1.2597	1.3380	1.5541	1.7935
1.2604	1.3433	1.5691	1.8240
1.2552	1.3423	1.5749	1.8394
1.2469	1.3372	1.5755	1.8475
1.2367	1.3291	1.5722	1.8508
1.2252	1.3185	1.5657	1.8502
1.2125	1.3506	1.5560	1.8465
1.1982	1.2900	1.5433	1.8396
1.1817	1.2710	1.5270	1.8298
1.1608	1.2460	1.5062	1.8167
2.6876	2.0828	1.6164	1.4248
1.2321	1.2842	1.3949	1.4711
1.2365	1.2987	1.4728	1.6706
1.2378	1.3010	1.4801	1.6866
1.2385	1.3026	1.4855	1.6990
1.2354	1.3015	1.4877	1.7069
1.2286	1.2968	1.4850	1.7071
1.2198	1.2895	1.4792	1.7032
1.2095	1.2709	1.4706	1.6961
1.1982	1.2684	1.4595	1.6863
1.1857	1.2547	1.4455	1.6735
1.1716	1.2384	1.4283	1.6575
1.1552	1.2182	1.4067	1.6396
1.1334	1.1901	1.3781	1.6124
2.7589	2.1461	1.6576	1.4494
1.2304	1.2782	1.3772	1.4401
1.2338	1.2940	1.4694	1.6801

1.2335	1.2938	1.4692	1.6793
1.2324	1.2928	1.4681	1.6786
1.2275	1.2888	1.4641	1.6743
1.2200	1.2825	1.4577	1.6675
1.2109	1.2741	1.4491	1.6584
1.2005	1.2639	1.4383	1.6471
1.1891	1.2518	1.4252	1.6332
1.1766	1.2378	1.4093	1.6165
1.1626	1.2211	1.3900	1.5963
1.1461	1.2003	1.3658	1.5714
1.1239	1.1707	1.3328	1.5393
2.7850	2.1700	1.6736	1.4585
1.4039	1.5840	2.0421	2.5461
1.4034	1.5837	2.0419	2.5459
1.4020	1.5828	2.0413	2.5453
1.3974	1.5797	2.0389	2.5432
1.3839	1.5692	2.0303	2.5354
1.3676	1.5547	2.0174	2.5232
1.3500	1.5376	2.0009	2.5070
1.3319	1.5184	1.9811	2.4873
1.3133	1.4973	1.9583	2.4641
1.2940	1.4743	1.9325	2.4374
1.2737	1.4489	1.9033	2.4071
1.2514	1.4203	1.8703	2.3727
1.2250	1.3863	1.8320	2.3335
2.4515	1.8898	1.4807	1.3243

BIBLIOGRAPHY

1. Heflinger, L. O., Wuerker, R. F., and Brooks, R. E., "Holographic Interferometry," Journal of Applied Physics, Vol. 37, No. 2, pp. 642-649, February 1966.

2. Brooks, R. E., Heflinger, L. O., and Wuerker, R. F., "9A9-Pulsed Laser Holograms," IEEE Journal of Quantum Electronics, Vol. EQ-2, No. 8, pp. 275-299, August 1966.

3. Matulka, R. D., "The Application of Holographic Interferometry to the Determination of Asymmetric Three-dimensional Density Fields in Free Jet Flow,", PhD Thesis, Naval Postgraduate School, June 1970.

4. Maldanado, C. D., Caron, A. P., and Olsen, H. N., "New Method for Obtaining Emission Coefficients from Emitted Spectral Intensities. Part I - Circularly Symmetric Light Sources," Journal of the Optical Society of America, Vol. 55, No. 10, pp. 1247-1254, Octover 1965.

5. Maldanado, C. D., and Olsen, H. N., "New Method for Obtaining Emission Coefficients from Emitted Spectral Intensities. Part II - Asymmetric Sources," Journal of the Optical Society of America, Vol. 56, No. 10, pp. 1305-1312, October 1966.

6. Jones, D. J., "Tables of Inviscid Supersonic Flow About Circular Cones at Incidence, Part I and II," Advisory Group for AeroSpace Research and Development, November 1969.

7. Liepmann, H. W., and Roshko, A., "Elements of Gas Dynamics," pp. 165, John Wiley and Sons, Inc., 1957.

8. Olsen, H. N., Maldanado, C. D., and Duckworth, G. D., "A Numerical Method for Obtaining Internal Emission Coefficients from Externally Measured Spectral Intensities of Asymmetric Plasmas," Journal of Quantum Spectroscopy and Radiation Transfer, Vol. 8, pp. 1419-1439, 1968.

9. Olsen, H. N., Maldanado, C. D., Duckworth, G. D., and Caron, A. P., "Investigation of the Interaction of an External Magnetic Field with an Electric Arc," Aero-Space Research Laboratories Report ARL66-0016, January, 1966.

10. Sullivan, J. B., "An Investigation of Three-Dimensionality in Holographic Interferometry," Masters Thesis, Naval Postgraduate School, 1968.

11. Matulka, R. D., and Collins, D. J., "The Application of Holographic Interferometry to the Determination of Asymmetric Three-Dimensional Density Field in Free Jet Flow."

12. Taylor, G. I., and Maccoll, J. W., "The Air Pressure on a Cone Moving at High Speed," Proceedings of the Royal Society, London, Series A, Vol. 139, 1933.

13. Stone, A. H., "On Supersonic Flow Past a Slightly Yawing Cone," Journal of Mathematics and Physics, Vol. 27, 1948.

14. Staff of the Computing Section of Analysis (under the direction of Zdenek Kopal), "Tables of Supersonic Flow Around Cones," Technical Report No. 1, NOrd Contract No. 9169, M.I.T., 1947.

15. Staff of the Computing Section, Centre of Analysis (under the direction of Zdenek Kopal), "Tables of Supersonic Flow Around Yawing Cones," Technical Report No. 3, NOrd Contract No. 9169, M.I.T., 1949.

16. Staff of the Computing Section (under the direction of Zdenek Kopal), "Tables of Supersonic Flow Around Cones of Large Yaw," NOrd Contract No. 8555 and 9169, M.I.T., 1949.

17. Jones, D. J., "Numerical Solutions of the Supersonic Flow Field for Conical Bodies in a Supersonic Stream," National Research Council of Canada National Aeronautical Establishment, Aeronautical Report LR-507, 1968.

18. Ladenburg, R., Van Voorhis, C. C., and Winckler, J., "Interferometric Study of Supersonic Phenomena - Part II," NAVORD Report 93-46, September, 1946.

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DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1 ORIGINATING ACTIVITY (Corporate author)

Naval Postgraduate School
Monterey, California 93940

2a. REPORT SECURITY CLASSIFICATION

Unclassified

2b. GROUP

3 REPORT TITLE

THE APPLICATION OF HOLOGRAPHIC INTERFEROMETRY TO THE DETERMINATION OF THE FLOW FIELD AROUND A RIGHT CIRCULAR CONE AT ANGLE OF ATTACK

4 DESCRIPTIVE NOTES (Type of report and inclusive dates)

Aeronautical Engineer's Thesis

5 AUTHOR(S) (First name, middle initial, last name)

Ravi Chandar Jagota, Lieutenant Commander, Indian Navy

6 REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
December 1970	150	18

8a. CONTRACT OR GRANT NO.	9a. ORIGINATOR'S REPORT NUMBER(S)
b. PROJECT NO.	
c.	9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
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13. ABSTRACT

The successful application of holography to the study of three dimensional flow fields due to phase objects has been reported in the literature. The present report extends this technique to the study of density fields around opaque bodies as would normally be encountered in wind tunnel experiments. The density field around a 10 degree half-angle cone at zero and ten degree angle of attack has been investigated in the Naval Postgraduate School supersonic wind tunnel. In these experiments, the finite fringe method for the production of interferograms has, for the first time, been applied to holographic interferometry. The density field obtained from the reduction of the interferograms was found to agree with that obtained from an analytical solution of the governing equations as reported by D. J. Jones in Reference 1. The computer program used for the reduction of the interferograms has been evaluated for the effect of the presence of the cone and the shock wave and has been found to yield stable and accurate results.

14 KEY WORDS	LINK A		LINK B		LINK C	
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1 ORIGINATING ACTIVITY *(Corporate author)*

Naval Postgraduate School
Monterey, California 93940

2a. REPORT SECURITY CLASSIFICATION

Unclassified

2b. GROUP

3. REPORT TITLE

The Application of Holographic Interferometry to the Determination of the Flow Field Around a Right Circular Cone at Angle of Attack

4. DESCRIPTIVE NOTES *(Type of report and, inclusive dates)*

Technical Report, NPS-57C071111A

5 AUTHOR(S) *(First name, middle initial, last name)*

Daniel J. Collins and Ravi C. Jagota

6. REPORT DATE

November 1971

7a. TOTAL NO. OF PAGES

153

7b. NO. OF REFS

18

8a. CONTRACT OR GRANT NO.

b. PROJECT NO.

c. Airtask No. A310310A/551A/1 R010-03-01

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